

ANALYSIS OF HADRONS ELASTIC INTERACTIONS AT HIGH ENERGIES

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The model of an inelastic overlap function (IOF) of elastic scattering of hadrons at high energies outgoes from a task of an IOF determination and versions of this model differ from each other by particular kind of (IOF). This model is based on the usage of S-channel condition of a unitarity. In this model absorptive nature of diffraction elastic scattering is obviously exhibited. According to this picture, the diffraction scattering is a shadow of absorption conditioned by existence of many inelastic reaction channels at high energies [1].

In the present approach initial is IOF is set, and the elastic overlap function (EOF) is determined from a condition of a unitarity at given IOF and EOF corresponds to the corrections of rescattering. IOF and EOF represent the contributions of inelastic and elastic reaction channels into the imaginary part of elastic scattering amplitude in consistent of unitarity condition.

Sometimes a reasonable question arises, why it is necessary to set IOF, or, we shall say an eikonal (eikonal of model), instead of scattering amplitude itself, because these values are inter connected among themselves. The answer can be presented by the following statement. At the absence of consequent theoretical approach it is possible to set that physical quantity, which is the most simple kind and carries a physical meaning, which can be a better base theoretically. For example, considered IOF has a more simple kind than scattering amplitude calculated with

its help [1], the square module of which gives an experimentally measured cross – sections of elastic scattering. IOF has a simple physical meaning: it causes a shadow effect of an absorption on diffraction elastic scattering. Though there are different simplified models for calculations of IOF, they are mainly based on the Feinman diagrams, which are suitable only for the direct description of experimental data [1, 2].

From experimental features of pp and $\bar{p}p$ elastic scattering discovered recently and till now can not be understood theoretically, we shall mark a diffraction picture with one fall in differential cross-section and evolution of this picture with energy, in particular, the transition of frame “minimum“-maximum” in “brachium”, and also increase of cross-sections and relations of material and imaginary parts of a scattering amplitude with definite energy [1-3]. With increase of energy the nucleon becomes more black, (so-called BEL-effect [1-3]). The profile function becomes close to stepwise, corresponding to the Fruassar-Marten limitation.

We used the IOF model earlier for the successful description of differential cross-section of pp and $\bar{p}p$ scatterings at separate energies [3, 4]. Since then old and obtained new experimental results were updated. In particular, recently have been appeared first measurements of differential cross-sections $\bar{p}p$ scatterings at $\sqrt{s} = 1800$ GeV on the Fermilab Tevatron collider [1, 2].

Earlier we systematically studied the differential cross-sections of elastic pp and $\bar{p}p$ -scattering in the wide region of energies $\sqrt{s} = 4-1800$ GeV and transferred momenta $0 \leq t \leq 10(\text{GeV})^2$ [1,2].

In the given work in the IOF approach frame the total cross-sections of pp -scattering at the energies from 6,2 up to 62 GeV and for $\bar{p}p$ - scattering in the energy region from 4,6 till 1800 GeV in the field of transferred momentum square $0 \leq t \leq 10(\text{GeV})^2$ were calculated. At these energies the ratio of real and imaginary parts of elastic scattering amplitude in the field of small scattering angles were calculated. The analysis show that values of $\sigma_{el}, \sigma_t, \sigma_{in}$ and δ are in a good agreement with existing experimental data.

The dependence of scattering amplitude from the transferred momentum is given by the Fourier – Bessel transformations of scattering amplitude $\varphi(s, \rho)$ in the representation of impact parameter ρ .

$$F(s, t) = i\sqrt{\pi} \int_0^\infty \varphi(s, \rho) I_0(\rho\sqrt{-t}) \rho d\rho, \quad (1)$$

where $I_0(x)$ - is Bessel function. Scattering amplitude is normalized so that differential cross – section of elastic scattering is equal to:

$$\frac{d\sigma}{dt} = |F(s, t)|^2, \quad (2)$$

and total cross – section $\sigma_t(s)$ according to the optical theorem is:

$$\sigma_t(s) = 4\sqrt{\pi} I_m F(s, 0). \quad (3)$$

The unitarity relationship in terms of overlap functions in the t – representation is:

$$4\sqrt{\pi} I_m F(s, t) = E(s, t) + G(s, t), \quad (4)$$

where $E(s, t)$ and $G(s, t)$ - EOF and IOF respectively. In the impact parameter representation unitarity condition can be presented as

$$2R_l \varphi(s, \rho) = \varphi_l(s, \rho) + \varphi_g(s, \rho), \quad (5)$$

where $\varphi_l \equiv |\varphi|^2$ and φ_g - EOF and IOF in the impact parameter representation. Physical solution of equation (5) has the following form:

$$\varphi(s, \rho) = 1 - \exp(2i\alpha(s, \rho))\sqrt{1 - \varphi_g(s, \rho)}, \quad (6)$$

Here α - is phase shift of pure elastic scattering. Solution (6) shows that elastic diffraction is a shadow of absorption, caused by the existing many open inelastic channels of reactions. Therefore, absorptive corrections or effects of rescattering (first term in (5)) play a very important role at the study of diffraction of elastic scattering. From (5) and (6) one can see, that only at small values of φ_g , and therefore at small $|\varphi|$, in (5) the squared term can be neglect.

Effect of rescattering can be easily see from the solution (6), if we put that $\alpha \cong 0$ and rewrite it as:

$$\varphi(s, \rho) \cong 1 - \sqrt{1 - \varphi_g(s, \rho)} = \frac{\varphi_g(s, \rho)}{2} + \frac{\varphi_g^2(s, \rho)}{2(1 + \sqrt{1 - \varphi_g(s, \rho)})^2}. \quad (6.1)$$

Relationship (5) can be written in the form:

$$0 \leq (1 - R_i \varphi(s, \rho))^2 + (I_m \varphi(s, \rho))^2 = 1 - \varphi_g(s, \rho) \leq 1. \quad (7)$$

IOF $\varphi_g(s, \rho)$ is limited by ranges

$$0 \leq \varphi_g(s, \rho) \leq 1, \quad (8)$$

and characterizes blackness and inelasticity.

We outgo from the following dependence of IOF from impact parameter

$$\varphi_g(s, \rho) \cong 2a \exp\left(-\frac{\rho^2}{2b_1}\right) - c\rho^2 \exp\left(-\frac{\rho^2}{b_1}\right); \quad (9)$$

here a, b_1, c - are model parameters, generally speaking, depending on the energy, where phase shift of pure elastic scattering we take in the form [5-7]

$$2\alpha(s, \rho) = d \exp\left(-\frac{\rho^2}{2b_2}\right), \quad (10)$$

where d and b_2 - are parameters, depending on the energy. At $c = 0$ (9) passes into the Van Hove model, taking into account only kinematics correlations of secondary particles (conservation rules of energy and momentum). Expression (9) at $c \neq 0$ is phenomenological and takes into account effects of the absorptive corrections, dynamic correlations of secondary particles and destructive interference conditioned by phases of matrix elements of inelastic processes. In the

(9) dependence from ρ is expressed over combination $\frac{\rho}{b_1}$, which by Fourier-Bessel

transformations leads to the $G(s, t)$ dependence of the transferred momentum via the $b_1 t$ value. If a and c parameters in (9) don't depend on the energy, then IOF has properties of geometrical scaling and we have $b_1(s)t$ as scaling variables.

At given $\varphi_g(s, \rho)$ and $\alpha(s, \rho)$ imaginary and real parts of scattering amplitude are determined by Fourier-Bessel integrals

$$I_m F(s, t) = \sqrt{\pi} \int_0^\infty (1 - \sqrt{1 - \varphi_g(s, \rho)}) I_0\left(\rho |t|^{1/2}\right) \rho d\rho + \\ 2\sqrt{\pi} \int_0^\infty \sqrt{1 - \varphi_g(s, \rho)} \sin^2 \alpha(s, \rho) I_0\left(\rho |t|^{1/2}\right) \rho d\rho, \quad (11)$$

$$R_i F(s, t) = \sqrt{\pi} \int_0^\infty \sqrt{1 - \varphi_g(s, \rho)} \sin 2\alpha(s, \rho) I_0\left(\rho |t|^{1/2}\right) \rho d\rho \quad (12)$$

At small transferred momentum the general behaviour of differential cross-section is determined by a collective phenomenon of a diffraction, which in turn is determined by a multiparticle unitarity condition. With increase of a transferred momentum there is a relevant dynamics of interplay of constituents of hadrons and at very large $-t$ the interplay of hadrons is reduced to interplay of their constituents. From the idealized point of view the greatest difficulties arise at intermediate transferred momentum. Different mechanisms play role in this

area. They should explain originating of frame such as «a dip-secondary maximum» and its turning into “«brachium»” at energies of a SPS-collider [1, 2, 5]. In the model of Islam [8] this frame is determined by an interference between a diffraction and hard scattering. In model of A. Donnachie and P.V. Landshoff [9] an interference takes place between single and double pomerons by the contributions at intermediate values $-t$, and at very large values $-t$ dynamics is determined by trimonial exchange. This model can explain various types of behavior of pp and $\bar{p}p$ scattering in the dip area, because the trimonial exchange is equivalent to effective odderon.

This model results in disappearance of dip in $\bar{p}p$ scattering and its conservation in pp -scattering at high energies. However it is necessary to mark that such phenomenon is not asymptotic, in accordance with the general theorem [1] at asymptotic energies pp and $\bar{p}p$ the scatterings should behave by an identical way. In accordance with formulae (9) and (12) IOF contains five parameters a , b_1 , b_2 , c and d , depending from energy. We will use our model for the description of experimental data on the pp – interaction at the energies $\sqrt{s} = 6, 2; 13,8; 19,4; 31; 53; 62$ GeV and on the $\bar{p}p$ - interaction at the $\sqrt{s} = 4, 6; 7,6; 9,8; 53; 546; 630$ and 1800 GeV [1,2].

Values of model parameters at these energies are shown in the Table 1. They are of a little difference from those used by us [1, 2, 7 earlier], which are shown in the Table 2. As demonstrate results of the present work, at low energies a parameter value is less than unit, and at high energies this value becomes more than unit. The unitarity equation for amplitude of elastic scattering with given as in the (9) IOF has three hardly distinguished solutions depending on a parameter value c : $0 \leq c \leq 1$, $c=0$, $c > 1$. The first solution corresponding to the $c < 1$ describes experiment on scattering on large angles better than in case of $c = 0$ corresponding to the model of uncorrelated Van – Hove jets. However, in case of $0 \leq c < 1$, though it is possible to receive a quantitatively correct parameter of an exponent, the factor before exponent is very small and it is impossible to describe quantitatively the experiment. The second solution ($c=1$) does not describe the scattering on large angles. The experimentally measured angular distributions at very high energies are adequately described by the third solution ($c > 1$). This solution corresponds to outcome of the theory of complex angular momenta. The strong difference of the first and third solutions is conditioned by the fact that in the first case the amplitude is determined by of constant signs series, while in second by alternating. Thus, there is a conformity between outcomes obtained within the framework of a condition of a unitarity for elastic scattering at large transferred momenta, and results of model of repeated exchange of reggeons. These cases pass one in to another at change of a parameter value (with $c < 1$ and $c > 1$) [1,2].

Table 1. Model parameters for pp and $\bar{p}p$ - interactions accepted in present paper.

	$\sqrt{s}$, GeV	A	b_1 , (GeV) ⁻²	b_2 , (GeV) ⁻²	C	D
pp	6,2	0,62	11,7	3,7	0,86	0,64
	13,8	0,77	9,9	3,5	0,99	-0,18
	19,4	0,76	10,2	3,2	1,01	-0,22
	31	0,81	10,2	3,2	1,01	-0,12
	53	0,70	12,2	4,1	1,05	-0,26
	62	0,69	12,9	4,4	1,07	-0,31
$\bar{p}p$	4,6	0,80	13,6	4,1	0,94	$1,3 \cdot 10^{-7}$
	7,6	0,81	11,7	3,4	1,08	-0,63
	9,8	0,92	10,1	2,7	1,003	-0,093
	53	0,73	12,1	3,8	1,05	-0,32
	546	0,78	16,8	6,8	1,22	-0,98
	630	0,66	18,1	6,6	1,04	-0,62
	1800	0,66	20,0	6,7	1,10	-1,40

As it is shown in the Table 1, the ratio of parameters $\frac{b_1}{b_2}$ is close to 3 at all energies for pp and $\bar{p}p$ - interactions. Unfortunately, it is difficult to extract from the Table 1 some well defined values relatively to other parameters.

In the Table 2 the characteristics of forward elastic scattering are presented computed with the help of values of parameters, reduced in the Table 1: total cross-sections and ratio of real and imaginary part of a scattering amplitude at energies in an interval of $4,5 \leq \sqrt{s} \leq 1800$ GeV.

Table 2. Calculated values of total cross-sections of σ_t, σ_{in} and σ_{el} and ratio of $\delta \cong \frac{R_l F(0)}{I_m F(0)}$ for pp and $\bar{p}p$ -interactions.

	$\sqrt{s}, \Gamma\text{ЭВ}$	σ_t, mb	σ_{in}, mb	σ_{el}, mb	$\delta \cong \frac{R_l F(0)}{I_m F(0)}$
pp	6,2	38,0	30,8	7,2	-0,17
	13,8	37,9	30,3	7,3	0,03
	19,4	37,7	30,6	7,1	0,04
	31	39,2	31,3	7,9	0,02
	53	41,7	34,3	7,1	0,06
	62	42,4	35,3	7,1	0,08
$\bar{p}p$	4,6	54,7	43,1	11,6	$-2 \cdot 10^{-8}$
	7,6	44,8	36,1	8,7	0,09
	9,8	45,0	35,0	10,0	$7 \cdot 10^{-3}$
	53	42,6	35	7,6	0,06
	546	62	49	13	0,25
	630	60,6	49,7	10,9	0,20
	1800	70,5	52,5	18	0,3

In Table 3 theoretical and experimental values of $\frac{\sigma_{el}}{\sigma_t}, \frac{\sigma_t}{b}, \frac{\sigma_{in}}{b}$ and $\frac{\sigma_{el}}{\sigma_{in}}$ at $\sqrt{s} = 546$ GeV in dependence of parameters c and δ^2 are shown.

Table 3.

C	δ^2	$\frac{\sigma_{el}}{\sigma_t}$ theor.	$\frac{\sigma_{el}}{\sigma_t}$ exp.	$\frac{\sigma_t}{b}$ theor.	$\frac{\sigma_t}{b}$ exp.	$\frac{\sigma_{in}}{b}$ theor.	$\frac{\sigma_{in}}{b}$ exp.	$\frac{\sigma_{el}}{\sigma_{in}}$ theor.	$\frac{\sigma_{el}}{\sigma_{in}}$ exp.
1	0,04	0,215	$0,215 \pm$	9,4	$10,5 \pm$	7,66	$8,2 \pm$	1,23	$1,28 \pm$
	0,07	0,240	0,005	9,5	0,3	7,79	0,3	1,21	0,06
1.1	0,04	0,202	$0,215 \pm$	9,3	$105 \pm$	7,43	$8,2 \pm$	1,21	$1,28 \pm$
	0,07	0,227	0,005	9,8	9,3	7,60	0,3	1,29	0,06

From this Table one can see that the theoretical and experimental relations of total cross-sections are well self-consistent, the calculated values of total cross-sections to slope parameter are in a good agreement within the limits of two standard deviations. The value $\delta = 0,20 - 0,27$ reduced in the table is in agreement with experimentally measured on $sp\bar{p}s$ - collider value for this relation $\delta = 0,24 \pm 0,04$.

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