Enhanced Alpha Decay Half-Life Determination in Super-Heavy Nuclei Using a New Potential

Aydın Yıldırım[a], İsmail Boztosun[b], Yasemin Küçük[b,c], Diana Gusseinova[b], Dilshod Alimov[d,e], Mardan Bakirov[d]

[a] *Proton Accelerator Facility, Nuclear Energy Research Institute (TENMAK-NUKEN), Ankara, 06980, Türkiye*
[b] *Physics Department, Akdeniz University, Antalya, 07058, Türkiye*
[c] *Turkish Accelerator and Radiation Laboratory (TARLA), Ankara, 06830, Türkiye*
[d] *Al-Farabi Kazakh National University, Al-Farabi, 71, Almaty, 050040, Kazakhstan*
[e] *Institute of Nuclear Physics, Ibragimova 1, Almaty, 050032, Kazakhstan*

Abstract

This paper systematically investigates alpha decay half-lives in the super-heavy nuclei region ($104 \leqslant Z \leqslant 118$), focusing on ground-state to ground-state transitions. It aims to enhance the accuracy of alpha decay half-life calculations by introducing a modified nuclear potential that includes surface deformation. Advanced techniques such as Bohr-Sommerfeld quantization conditions and Langer modifications are used to develop an effective potential for the interaction between the daughter nucleus and the alpha particle. The Woods-Saxon potential, modified with a Gaussian term to account for surface deformation, is employed in numerical calculations of alpha decay half-lives. These calculated half-lives are compared with experimental data and the previous studies, showing significantly improved agreement. The modified potential offers greater precision in modeling the alpha decay process, particularly for even-even super-heavy nuclei. This research introduces a novel approach to alpha decay studies, enhancing accuracy and applicability. The modified nuclear potential with surface deformation proves to be a valuable tool in nuclear physics, providing more reliable alpha decay half-life estimates. The implications of this work extend to the creation of new super-

Email addresses: aydinyldrm@gmail.com (Aydın Yıldırım), boztosun@akdeniz.edu.tr (İsmail Boztosun), ykucuk@akdeniz.edu.tr (Yasemin Küçük), gusseinovadi@gmail.com (Diana Gusseinova), d_alimov@inp.kz (Dilshod Alimov), b.mardan@mail.ru (Mardan Bakirov)

heavy nuclei and astrophysical processes. Further research can build upon these findings to explore a broader range of nuclear systems and advance our understanding of nuclear physics.

Keywords: Alpha Decay, Deformation, Half-Life, WKB method, Woods-Saxon

1. Introduction

Alpha decay is a fundamental phenomenon in nuclear physics that continues to attract attention for further theoretical and experimental studies. It serves not only as a critical component of nuclear physics but also as a manifestation of quantum mechanical tunneling [1–4]. Research has revealed that alpha particles may exist within nuclei prior to their decay, leading to the development of cluster models in nuclear physics [5, 6]. In these models, heavier nuclei, beyond the alpha particle, are also preformed within the nucleus, and these clusters undergo decay via quantum tunneling, similar to alpha particles. Alpha decay and cluster models enhance our understanding of nuclear structure and behavior.

Determining the interaction potential between alpha particles (or clusters) and the nucleus holds significance not only from a theoretical perspective but also for its potential role in creating new Super-Heavy Nuclei (SHN) within accelerator facilities [7–11]. Alpha decay is intrinsically tied to the penetration of alpha particles through the nuclear potential barrier. Consequently, multiple models have been developed to explain this interaction. The half-life of parent nuclei can be analytically calculated using the nuclear potential and alpha decay energy, as proposed by Gamow, 1928 [1]. This provides a means to assess the effectiveness of the nuclear potential [12–14]. A closer agreement between calculated and experimental half-lives signifies greater accuracy in the nuclear potential. Achieving a nuclear potential that accurately matches experimental half-life results across a range of atomic numbers 'Z' would not only advance our understanding of the nucleus but also provide insights into cluster models, alpha decay, and the synthesis of new SHN in accelerator facilities.

In addition to its theoretical implications, research focused on constructing a successful nuclear potential and unraveling the intricacies of alpha decay holds the potential to generate new heavy and super-heavy nuclei, with significant implications for nuclear medicine studies. While known alpha-emitting

nuclei are limited in number, only a select few are employed for medical applications. Targeted alpha therapy (TAT) stands out as a prominent nuclear therapy, with its short-range effectiveness within cells or tissues and high linear energy transfer (LET) capabilities. However, the current scarcity of alpha emitters restricts its applicability to specific medical conditions [15]. Expanding the range of alpha emitters can extend the scope of TAT across a broader spectrum of medical applications. The half-life of alpha emitters, whether short, medium, or long, directly correlates with the types of diseases for which they can be employed. In this context, the quest for alpha emitters with varying half-lives is pivotal. Moreover, alpha emitters with stable daughters are particularly desirable in nuclear medicine, as their emissions post-treatment should not jeopardize healthy tissue by emitting harmful radiation [16]. Consequently, research aimed at synthesizing new nuclei bears significant relevance.

Another intriguing aspect of new nucleus synthesis, particularly superheavy nuclei, pertains to its implications in astrophysics [17–19]. The r-process (rapid neutron capture process) nucleosynthesis in astrophysics remains a topic of great interest, yet it is still not comprehensively understood, representing a captivating subject in both nuclear physics and nuclear astrophysics. The fission processes involved in the formation of super-heavy elements are believed to initiate subsequent r-processes [19]. As these processes are exceedingly complex, there is a growing body of research focused on unraveling their complexities. However, the success of these studies is heavily reliant on the accuracy of the models employed. Hence, the development of more precise models promises to substantially enhance our comprehension of these processes.

Alpha decay, occurring through quantum tunneling, is intricately linked to the nuclear potential. However, a single potential cannot sufficiently describe all nuclei, leading to the development of various models specific to different atomic number regions [20–22]. Among these models, the Woods-Saxon potential is recognized as one of the most successful, although it has its limitations when describing deformed nuclei, a category that includes the majority of nuclei emitting alpha particles [7, 8, 11, 23]. In view of alpha decay predominantly occurring at the surface of the nucleus, this study introduces a Gaussian deformation term to the Woods-Saxon potential, resulting in a surface-deformed potential. In a prior investigation, it is successfully demonstrated the effectiveness of this modified potential in predicting the behavior of ten selected nuclei [24]. Building upon this foundation, the cur-

rent research extends to encompass the range of $104 \leqslant Z \leqslant 118$. It is our aim to study and improve the accuracy of alpha decay in the super-heavy nuclei region, focusing on ground-state to ground-state transitions using the Wentzel-Kramers-Brillouin (WKB) method and Bohr-Sommerfeld quantization conditions [25].

The organization of this paper is as follows. In Section 2, we delve into the theoretical aspects of our study. Section 3 presents our numerically calculated results, followed by a discussion of these outcomes in comparison with experimental data and previous work. In the final section, we provide a summary of our study and draw conclusions from our findings.

2. The Model

2.1. Modified Woods-Saxon Potentials

In the context of alpha decay, the interaction potential between the alpha particle and the daughter nucleus is of paramount importance. The penetration probability of the alpha particle is inherently related to the interaction potential, the decay energy Q_α, and the classical turning points determined by the equation $V_{eff}(r)=Q_\alpha$. These turning points, designated as r_1, r_2, and r_3, exhibit distinct associations. The outermost turning point r_3 is only dependent on the well-known centrifugal and Coulomb potentials [22]. The inner turning point r_2 is strongly related to the attractive nuclear potential. The nuclear potential is less precisely characterized and necessitates adjustments across different nuclei. The variation of the nuclear potential predominantly changes the r_2 turning point, especially at the surface of the effective potential. Therefore, the variation in the nuclear potential around the surface of the effective potential should be carefully examined. Since most alpha-emitting nuclei are deformed, the deformed nuclear potential is typically represented by the deformation of the nuclear radius [5, 26]. However, the deformation of the nuclear radius induces a new degree of freedom, which is the orientation angle. In contrast to previous studies, we employ a method that deforms the nuclear potential in proximity to the nuclear surface rather than the radius itself. This approach involves modifying the depth of the potential near the surface. To achieve a comprehensive understanding, two different forms of the deformed potential were utilized, and the results were compared with experimental data. The methodology underlying this approach is presented in detail in this section.

In our study, we describe the modified Woods-Saxon potentials by incorporating deformation terms. To comprehensively analyze these potentials, we consider two specific forms of the deformation term: a Gaussian function and its squared form. The modified Woods-Saxon potentials are defined in Equations (1) and (2) as follows:

$$V_{G1}(r) = -V_0 \left(\frac{1}{1 + e^{\frac{r-R}{a}}} + e^{-(\frac{r-R}{r'})^2} \right) \tag{1}$$

$$V_{G2}(r) = -V_0 \left(\frac{1}{1 + e^{\frac{r-R}{a}}} + (e^{-(\frac{r-R}{r'})^2})^2 \right) \tag{2}$$

In Eq.(1) and (2), the variables and parameters are defined as follows: λ is the normalization parameter, $R = 1.07A_d^{1/3}$ represents the nuclear radius, A_d is the mass number of the daughter nucleus, V_0 denotes the depth of the nuclear potential, $a = 0.70$ fm stands for the diffusion parameter, and r' is a constant in length dimension. The selection of the constant r' is discussed in Section 3.

2.2. Centrifugal and Coulomb Potentials

The effective potential between the alpha particle and the daughter nucleus is expressed as:

$$V_{eff}(r) = V_N(r) + V_L(r) + V_C(r). \tag{3}$$

where the first term is the nuclear potential which is represented by Eq. (1) or (2). The second term in Eq.(3) is the well-known centrifugal potential with Langer modifications in the following form [27] :

$$V_L(r) = \frac{\hbar^2 \left(L + \frac{1}{2}\right)^2}{2\mu r^2} \tag{4}$$

The L term in Eq.(4) shows the transferred angular momentum while $\mu = \frac{A_\alpha A_d}{A_\alpha + A_d}$ is the reduced mass of the alpha particle and daughter nucleus. A_α is the mass number of the alpha particle. It must be noted that we are only interested in the favored decay of super-heavy nuclei ($L = 0$). The Coulomb potential, which is the last term in Eq.(3), can be expressed in two parts as,

$$V_C(r) = \begin{cases} \frac{Z_\alpha Z_d e^2}{r} & , r \geq R_C \\ \frac{Z_\alpha Z_d e^2}{2R_C} \left[3 - \left(\frac{r}{R_C} \right)^2 \right] & , r < R_C \end{cases} \tag{5}$$

In Eq.(5), Z_α and Z_d represent the charges of the alpha and the daughter nucleus, respectively. Due to a discontinuity at $r = R_C$ in Eq. (5), we employ the following approximation [13, 28],

$$\widetilde{V}_C(r) = \frac{Z_\alpha Z_d e^2}{r}\left[1 - e^{-\varphi r - \frac{1}{2}(\varphi r)^2 - 0.35(\varphi r)^3}\right] \tag{6}$$

where $\varphi = \frac{3}{2R_C}$. As mentioned above, R_C is equal to $r_0 \times A_d^{1/3}$ with $r_0 = 1.07$.

2.3. Visualization of Potentials and Alpha Decay Half-Life

To provide a visual representation of the effective potentials in Eq. (3), Fig. (1) illustrates the Woods-Saxon potential and the surface-deformed potentials. In this figure, the red dashed line corresponds to the Woods-Saxon (WS) potential, the blue line represents the WS + Gauss potential (WS+G1), and the black line showcases the WS potential combined with the square of the Gaussian term (WS+G2). Both modified potentials exhibit deformations near the surface. These deformations play a critical role in the alpha decay calculations as discussed in [24].

The potentials representing the Fig.(1) have three turning points for a given alpha emitter. In order to find these three turning points namely r_1, r_2 and r_3, one must find the roots of $V_{eff}(r) = Q_\alpha$ equation [24, 26]. The depth of the nuclear potential V_0 can be eliminated using the Bohr-Sommerfeld quantization condition for each decay system [22, 29]. The Bohr-Sommerfeld quantization condition is given as,

$$\int_{r_1}^{r_2} dr \left(\frac{2\mu}{\hbar^2}\left[Q_a - V_{eff}(r)\right]\right)^{1/2} = (G - L + 1)\frac{\pi}{2} \tag{7}$$

In Eq.(7), Q_α represents the decay energy, and G represents the global quantum number. To determine global quantum number, we employ Wildermuth's quantum rule [30];

$$G = 2n + L = \sum_{i=1}^{4}(2n_i + l_i) = \sum_{i=1}^{4} g_i \tag{8}$$

where g_i represents the oscillator quantum numbers of the α cluster. The quantum number G can be taken as follows: $G = 22$ for $N > 126$, $G = 20$ for $82 < N \leqslant 126$ and $G = 18$ for $N \leqslant 82$ [29, 31, 32]. The half-life of alpha decay can be calculated using the formula;

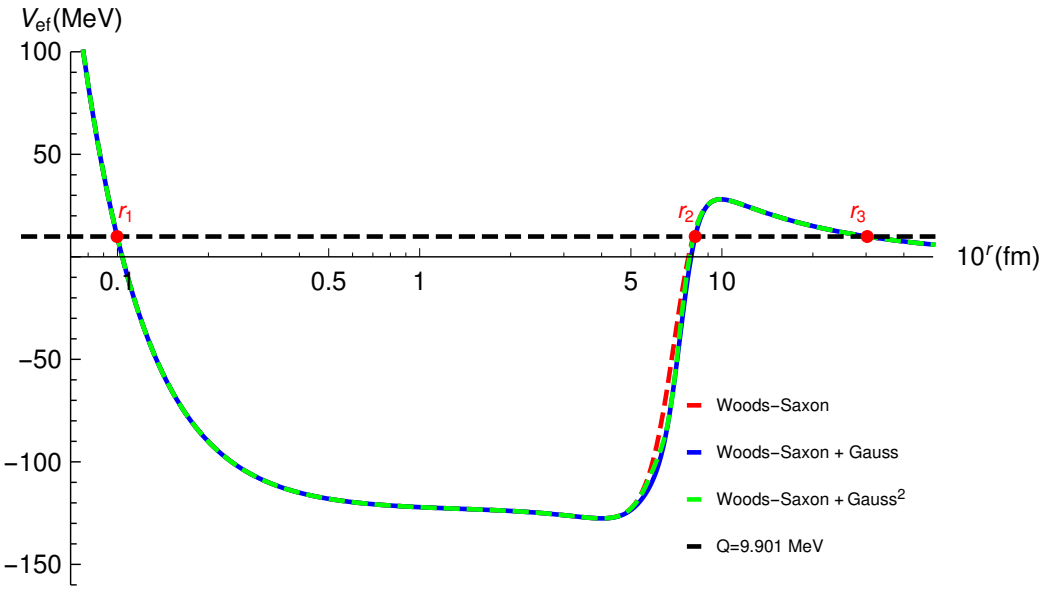

Figure 1: Plot of the effective potentials for three cases of the nuclear potentials: The Woods-Saxon, Woods-Saxon + Gauss (V_{G1}), and Woods-Saxon + Gauss^2 (V_{G2}) with $^{260}\mathrm{Sg} \rightarrow {}^{256}\mathrm{Rf} + \alpha$. The parameters of the given case are $Q_\alpha = 9.901$ MeV. The red points show the values of turning points r_1, r_2, and r_3.

$$T_{1/2} = \hbar \frac{\ln 2}{\Gamma} \tag{9}$$

where Γ represents the alpha decay width and can be expressed as [29, 33, 34];

$$\Gamma = PF \frac{\hbar^2}{4\mu} S. \tag{10}$$

F and S represent the normalization factor, and transition probability, respectively. The preformation factor P is given by $P = 1.0$ for even-even nuclei, $P = 1.0$ for odd A nuclei, and $P = 0.35$ for odd-odd nuclei [31, 32, 35]. The normalization factor F, and transition probability S can be expressed as [13];

$$F \int_{r_1}^{r_2} dr \frac{1}{2k(r)} = 1, \tag{11}$$

and

$$S = \exp\left(-2 \int_{r_2}^{r_3} dr k(r)\right), \tag{12}$$

where $k(r)$ represents the wave number between the turning points r_1 and r_2 and is given by $k(r) = \sqrt{\frac{2\mu}{\hbar^2}|V_{eff}(r) - Q_\alpha|}$.

3. Results and Discussion

As elaborated in the preceding section, using the WKB method and Bohr-Sommerfeld condition, we calculated the alpha decay half-lives of the selected nuclei. To account for the deformation effects of the super-heavy nuclei, we introduced two surface deformation terms to the Woods-Saxon potential, as exemplified in Figure (1). This study presents the alpha decay half-life calculations of 50 super-heavy nuclei[13, 36, 37].

To evaluate our findings, we compared them to the experimental and theoretical half-life values obtained from previous studies Coban et al. [13], Cheng et al. [36], Hosseini and Hassanabadi [37]. The numerical estimates for the half-lives, using both modified potentials, are presented in Tables 1, 3 and 5. The tables contain seven columns. The first column displays the parent nuclei that undergo alpha decay. The second column lists the Q_α value of the related decay. The third column shows the experimentally measured half-lives for the alpha decay. The fourth and fifth columns show the half-lives in seconds calculated with modified potentials in Eqs. (1) and (2). Finally, the sixth and seventh columns represent the deviations of half-lives resulting from modified potentials compared to experimental ones. It should be noted that the Q_α values used in this study are based on the most recent data set from Wang et al. [38], which may be different from those used in previous studies.

Table 1 shows the half-lives for 20 super-heavy nuclei. There are 5 even-even, 9 even-odd, 3 odd-even, and 3 odd-odd nuclei in this data set. It is noteworthy that while there are substantial differences for two nuclei, such as $^{268}Mt \rightarrow {}^{264}Bh + \alpha$, the modified potentials show success in explaining alpha decay for many of the nuclei studied. In fact, in several cases our results are in close agreement with experimental values, a demonstration of the effectiveness of our approach.

To further evaluate our results, we compared them not only with experimental values but also with previous theoretical studies, using a deviation analysis for the nuclei studied. In order to make such a comparison, we calculated the root mean square (RMS) deviations for each set of cases using

Table 1: The table shows the result of this study and the experimental values of the corresponding decay [13]. The Q_α values in the second column are taken from Wang et al. [38]. The sixth and seventh columns show the deviations from the experimental values.

Nuclei	Q_α [MeV] [38]	$T_{1/2}^{Exp}$ [s] [13]	$T_{1/2}^{G1}$ [s]	$T_{1/2}^{G2}$ [s]	$\log\left(\frac{T_{1/2}^{G1}}{T_{1/2}^{Exp}}\right)$	$\log\left(\frac{T_{1/2}^{G2}}{T_{1/2}^{Exp}}\right)$
^{260}Sg	9.901	9.12×10^{-3}	1.75×10^{-2}	2.10×10^{-2}	0.282	0.362
^{261}Sg	9.714	7.2×10^{-2}	9.12×10^{-2}	1.10×10^{-1}	0.103	0.183
^{263}Sg	9.400	1.2×10^{-1}	6.67×10^{-1}	8.07×10^{-1}	0.745	0.827
^{265}Sg	9.050	2.4×10^{1}	7.05×10^{0}	8.56×10^{0}	-0.532	-0.448
^{266}Sg	8.800	2.6×10^{1}	2.54×10^{1}	3.09×10^{1}	-0.011	0.074
^{261}Bh	10.500	3.39×10^{-3}	1.81×10^{-3}	2.18×10^{-3}	-1.271	-1.192
^{264}Bh	9.860	4.44×10^{-1}	1.30×10^{-1}	1.57×10^{-1}	-0.530	-0.448
^{265}Bh	9.660	9.4×10^{-1}	2.64×10^{-1}	3.19×10^{-1}	-0.552	-0.469
^{266}Bh	9.430	1.0×10^{0}	2.01×10^{0}	2.44×10^{0}	0.303	0.387
^{267}Bh	9.230	1.7×10^{1}	4.48×10^{0}	5.45×10^{0}	-0.579	-0.494
^{264}Hs	10.591	5.4×10^{-4}	1.30×10^{-3}	1.57×10^{-3}	0.382	0.462
^{265}Hs	10.470	5.8×10^{-4}	4.19×10^{-3}	5.06×10^{-3}	0.859	0.940
^{266}Hs	10.346	2.3×10^{-3}	5.02×10^{-3}	6.06×10^{-3}	0.339	0.421
^{267}Hs	10.038	7.4×10^{-2}	5.19×10^{-2}	6.29×10^{-2}	-0.154	-0.071
^{269}Hs	9.270	7.1×10^{0}	7.62×10^{0}	9.30×10^{0}	0.031	0.117
^{268}Mt	10.770	7.0×10^{-2}	2.62×10^{-3}	3.16×10^{-3}	-1.427	-1.345
^{269}Ds	11.510	2.7×10^{-4}	6.21×10^{-5}	7.48×10^{-5}	-0.638	-0.558
^{270}Ds	11.117	1.0×10^{-4}	2.79×10^{-4}	3.37×10^{-4}	0.446	0.528
^{271}Ds	10.870	6.2×10^{-4}	1.73×10^{-3}	2.10×10^{-3}	0.446	0.529
^{273}Ds	11.370	1.1×10^{-4}	1.12×10^{-4}	1.35×10^{-4}	0.007	0.089

the formula as,

$$\delta = \sqrt{\frac{1}{N-1}\sum_{n=1}^{N}\left[\log_{10}\left(T_{\mathrm{n},\,1/2}^{\mathrm{calc}}\right) - \log_{10}\left(T_{\mathrm{n},\,1/2}^{\mathrm{exp}}\right)\right]^2}. \tag{13}$$

where N, $T_{\mathrm{n},\,1/2}^{\mathrm{calc}}$, and $T_{\mathrm{n},\,1/2}^{\mathrm{exp}}$ are the number of decay systems, the calculated and experimental alpha decay half-lives, respectively. The RMS deviations for this study and previous research are presented in Table 2. The second and third rows display the results of this study. RMS deviations are calculated for various sets of cases, including total, even-even, even-odd, odd-even, and odd-odd nuclei, taking into account the two modified potentials employed in this study.

The deviation analysis for this dataset clearly shows that both modified potentials are better than the previous study for the alpha decay half-life

Table 2: RMS deviations for this and previous study [13]. The second and third rows show the results of this study.

Model	Total	even - even	even - odd	odd - even	odd - odd	Ref.
WS+G1	0.625	0.367	0.527	1.062	1.098	This Study
WS+G2	0.619	0.449	0.550	0.971	1.039	This Study
Def. Woods-Saxon	0.639	0.252	0.533	1.335	1.005	[13]

calculations of super-heavy nuclei. Looking more closely at the table, one can quickly see that the previous study works better for even-even nuclei. For even-odd nuclei, the WS+G1 potential is slightly better than the previous work, while the WS+G2 potential is slightly worse. The modified potentials showed their success in odd-even nuclei giving way better half-lives. Again, the results for odd-odd nuclei are very close, but slightly worse than previous work.

The second dataset is taken from Cheng et al. [36] and contains a total of 24 nuclei. As we mentioned before, 5 of these nuclei are already presented in Table 1, but in order to make a complete comparison with the previous study, all nuclei are kept in the table. In contrast to Table 1, this table contains only even-even nuclei. Although Table 3 looks like it contains much more deviation compared to Table 1, it also contains much better results for the rest of the nuclei for both the modified potentials. The RMS values of these data are calculated as follows: $\delta = 0.610$ for WS+G1 potential, $\delta = 0.873$ for WS+G2 potential, and $\delta = 1.168$ for an improved generalized liquid drop model in Cheng et al. [36], which shows a huge improvement in the calculation of alpha decay half-lives. Note that since this table consists of even-even nuclei only, it is not separated for total, even-even, even-odd etc.

It should be noted that since Table 3 contains only even-even nuclei, we were able to see a trend for the r' parameter in the Eq.(1) and Eq.(2). The adjustment of this parameter leads to results very close to the experimental decay half-lives. By systematically varying the r' parameter for each nucleus and selecting the value that best matches the experimental data, we found a consistent trend. Specifically, when we decrease the parameter r' from $r' = 4$ to $r' = 1$ for the parent nuclei with proton number $Z = 104$ to $Z = 110$, we obtain half-life values closest to the experimental ones. This trend is also observed for the nuclei with proton number $Z = 112$ to $Z = 118$. This systematic adjustment for each even-even nucleus revealed a relationship

Table 3: The table shows the result of this study and the experimental values of the corresponding decay [36]. The Q_α values in the second column are taken from Wang et al. [38]. The sixth and seventh columns show the deviations from the experimental values.

Nuclei	Q_α [MeV] [38]	$T_{1/2}^{Exp}$ [s] [36]	$T_{1/2}^{G1}$ [s]	$T_{1/2}^{G2}$ [s]	$\log\left(\frac{T_{1/2}^{G1}}{T_{1/2}^{Exp}}\right)$	$\log\left(\frac{T_{1/2}^{G2}}{T_{1/2}^{Exp}}\right)$
^{254}Rf	9.210	2.32×10^{-5}	1.29×10^{-3}	1.70×10^{-2}	1.746	2.864
^{256}Rf	8.926	6.67×10^{-3}	8.94×10^{-3}	1.15×10^{-1}	0.127	1.237
^{258}Rf	9.196	1.38×10^{-2}	1.22×10^{-3}	1.62×10^{-2}	-1.052	0.069
^{260}Rf	8.900	2.10×10^{-2}	9.27×10^{-3}	1.20×10^{-1}	-0.355	0.758
^{262}Rf	8.490	2.50×10^{-1}	1.90×10^{-1}	2.40×10^{0}	-0.119	0.982
^{258}Sg	9.670	2.70×10^{-3}	2.76×10^{-3}	1.53×10^{-2}	0.009	0.754
^{260}Sg	9.901	4.95×10^{-3}	5.94×10^{-4}	3.33×10^{-3}	-0.921	-0.172
^{262}Sg	9.600	1.09×10^{-2}	3.77×10^{-3}	2.10×10^{-2}	-0.462	0.284
^{264}Sg	9.210	4.70×10^{-2}	4.88×10^{-2}	2.69×10^{-1}	0.016	0.757
^{266}Sg	8.800	3.90×10^{-1}	8.80×10^{-1}	4.80×10^{0}	0.353	1.090
^{264}Hs	10.591	5.40×10^{-4}	6.97×10^{-4}	1.23×10^{-3}	0.111	0.356
^{266}Hs	10.346	3.02×10^{-3}	2.68×10^{-3}	4.73×10^{-3}	-0.052	0.195
^{268}Hs	9.760	3.80×10^{-1}	9.28×10^{-2}	1.64×10^{-1}	-0.612	-0.364
^{270}Hs	9.070	9.00×10^{0}	9.74×10^{0}	1.73×10^{1}	0.034	0.285
^{270}Ds	11.117	2.05×10^{-4}	2.79×10^{-4}	3.37×10^{-4}	0.134	0.216
^{280}Ds	9.710	1.10×10^{1}	8.91×10^{-1}	1.09×10^{0}	-1.092	-1.002
^{282}Cn	10.150	9.00×10^{-4}	9.51×10^{-4}	1.22×10^{-2}	0.024	1.131
^{284}Cn	9.670	1.04×10^{-1}	2.17×10^{-2}	2.69×10^{-1}	-0.680	0.412
^{284}Fl	10.700	3.30×10^{-3}	3.79×10^{-3}	1.43×10^{-2}	0.060	0.636
^{286}Fl	10.360	1.40×10^{-1}	2.80×10^{-2}	1.05×10^{-1}	-0.698	-0.123
^{288}Fl	10.076	7.50×10^{-1}	1.61×10^{-1}	6.02×10^{-1}	-0.669	-0.095
^{290}Lv	11.000	8.30×10^{-3}	6.54×10^{-3}	1.71×10^{-2}	-0.104	0.313
^{292}Lv	10.791	1.30×10^{-2}	2.08×10^{-2}	5.45×10^{-2}	0.205	0.622
^{294}Og	11.870	1.15×10^{-3}	9.84×10^{-4}	1.22×10^{-3}	-0.068	0.025

between the r' values and the nuclear properties, which we found to be crucial for accurate half-life predictions. The change in the potential due to this parameter adjustment is shown in Fig. (2). According to Fig. (2), the adjustment of r' affects not only the shape of the surface of the potential but also the depth of the potential before the second turning point, resulting in an improvement in the alpha decay calculation. The transition probability S in Eq. (12) is the dominant parameter, a multiplier of the power of ten, in determining the alpha decay half-life. It depends on the intersection of the effective potential with the alpha decay energy between the turning points r_2 and r_3 for a given alpha emitter [12]. The normalization factor F in Eq. (11) changes the half-life very little compared to S and depends on the intersection

of the effective potential with the alpha decay energy between the turning points r_1 and r_2 for a given alpha emitter. The values of the parameter r' for each nucleus are listed in Table 4.

Table 4: The table presents the values of r' based on the proton number of the nucleus.

Z	102	104	106	108	110	112	114	116
r'	4	3	1.5	1	4	2.5	2	1

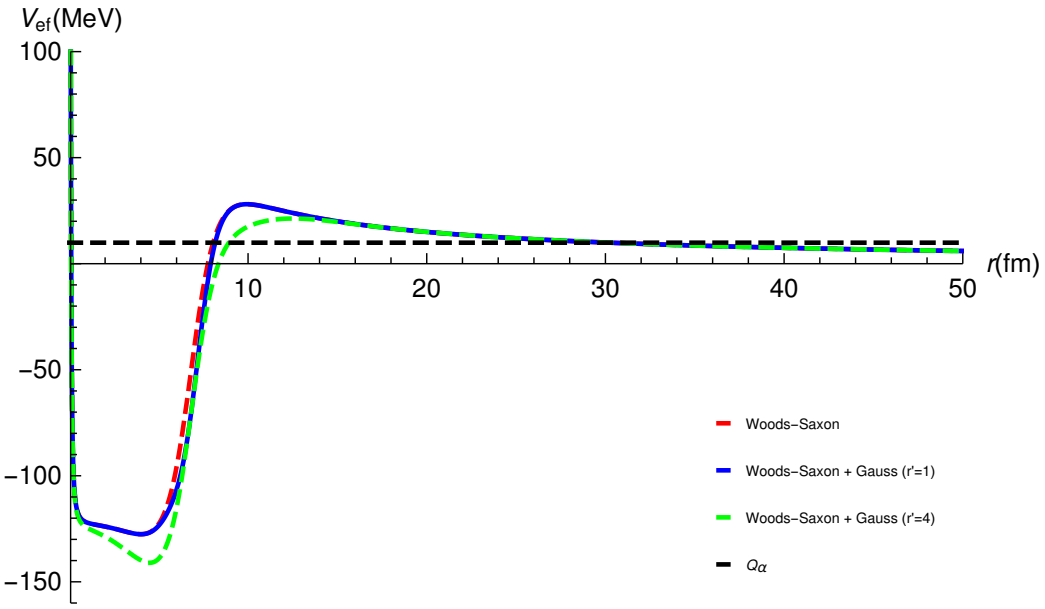

Figure 2: Plot of three different potentials for the α-decay of $^{260}\mathrm{Sg} \rightarrow {}^{256}\mathrm{Rf} + \alpha$ reaction. The effect of r' parameter can be seen clearly in the plot.

The last dataset is shown in Table 5 and is taken from Hosseini and Hassanabadi [37]. This dataset contains a total of 18 super-heavy nuclei, 11 of them are not already given in Tables 1 and 3. The 7 nuclei that coincide with Table 3 are even-even, while the remaining 11 nuclei are even-odd. The authors analyzed the given nuclei with two empirical formulas, namely Viola-Seaborg and Royer and also with a new analysis in the Brown formula (mB1 and mB2) [37, 39]. The RMS values for Table 5 are $\delta = 0.530$ for WS+G1 potential and $\delta = 0.584$ for WS+G2 potential. The four empirical formulas analyzed in Hosseini and Hassanabadi [37] give $\delta = 0.781$ for Viola-Seaborg,

Table 5: The table shows the result of this study and the experimental values of the corresponding decay [37]. The Q_α values in the second column are taken from Wang et al. [38]. The sixth and seventh columns show the deviations from the experimental values.

Nuclei	Q_α [MeV] [38]	$T_{1/2}^{Exp}$ [s] [37]	$T_{1/2}^{G1}$ [s]	$T_{1/2}^{G2}$ [s]	$\log\left(\frac{T_{1/2}^{G1}}{T_{1/2}^{Exp}}\right)$	$\log\left(\frac{T_{1/2}^{G2}}{T_{1/2}^{Exp}}\right)$
^{267}Rf	7.890	8.28×10^{3}	9.34×10^{3}	1.14×10^{4}	0.052	0.139
^{271}Sg	8.750	1.44×10^{2}	5.22×10^{1}	6.37×10^{1}	-0.440	-0.354
^{275}Hs	9.450	6.67×10^{0}	1.81×10^{0}	2.21×10^{0}	-0.566	-0.479
^{279}Ds	10.110	1.80×10^{-1}	1.17×10^{-1}	1.43×10^{-1}	-0.187	-0.100
^{281}Ds	9.470	9.60×10^{0}	7.31×10^{0}	9.00×10^{0}	-0.118	-0.028
^{282}Cn	10.150	5.00×10^{-4}	9.51×10^{-4}	1.22×10^{-2}	0.279	1.386
^{283}Cn	9.890	4.00×10^{0}	2.22×10^{0}	2.74×10^{0}	-0.256	-0.165
^{284}Cn	9.670	1.01×10^{-1}	2.17×10^{-2}	2.69×10^{-1}	-0.667	0.425
^{285}Cn	9.390	3.40×10^{1}	6.41×10^{1}	7.95×10^{1}	0.276	0.369
^{286}Fl	10.360	1.60×10^{-1}	2.8×10^{-2}	1.05×10^{-1}	-0.756	-0.181
^{287}Fl	10.170	5.10×10^{-1}	1.67×10^{0}	2.07×10^{0}	0.515	0.608
^{288}Fl	10.076	8.00×10^{-1}	1.61×10^{-1}	1.01×10^{0}	-0.697	0.100
^{289}Fl	9.950	2.70×10^{0}	6.59×10^{0}	8.19×10^{0}	0.387	0.482
^{290}Lv	11.000	1.50×10^{-2}	6.54×10^{-3}	1.71×10^{-2}	-0.361	0.056
^{291}Lv	10.890	6.30×10^{-3}	8.86×10^{-2}	1.10×10^{-1}	1.148	1.242
^{292}Lv	10.791	1.80×10^{-2}	2.08×10^{-2}	5.45×10^{-2}	0.064	0.481
^{293}Lv	10.680	5.30×10^{-2}	2.90×10^{-1}	3.61×10^{-1}	0.739	0.834
^{294}Og	11.870	1.80×10^{-3}	9.84×10^{-4}	1.22×10^{-3}	-0.263	-0.170

$\delta = 0.966$ for Royer, $\delta = 0.568$ for mB1 and $\delta = 0.571$ for mB2. The complete analysis of the RMS values for this dataset is given in Table 6.

The RMS values show that both WS+G1 and WS+G2 are better than Viola-Seaborg and Royer while mB1 and mB2 are better than WS+G2 overall. For the even-even case, both modified potentials are only better than Royer. For the even-odd case, both WS+G1 and WS+G2 are better than all four empirical formula models. This result also indicates that the modified potentials investigated in this study, particularly WS+G1 have a wider applicability than the previous work.

The investigation of alpha decay for a range of nuclei reveals significant findings. The modified potential approach, as demonstrated by the results, provides valuable insights into the behavior of alpha decay in various nuclei. Notably, while some deviations exist for specific cases, the modified potential offers a promising framework for explaining alpha decay processes. Exploring the alpha decay half-lives of 24 super-heavy nuclei in Table 3 uncovers interesting trends. Our results not only align closely with experimental data but,

Table 6: RMS deviations for this study and the previous study [37].

Model	Total	even - even	even - odd	Ref.
WS+G1	0.530	0.545	0.546	This Study
WS+G2	0.584	0.634	0.582	This Study
Viola-Seaborg	0.781	0.396	0.971	[37]
Royer	0.966	1.286	0.770	[37]
mB1	0.568	0.430	0.661	[37]
mB2	0.571	0.401	0.677	[37]

in most instances, surpass previous studies. This highlights the potential of our approach for modeling alpha decay in the realm of even-even super-heavy nuclei. The calculation of RMS deviations further supports the accuracy of our results. The minimal RMS deviations for a majority of cases demonstrate the reliability of our approach in explaining alpha decay.

4. Conclusion

This paper introduces a novel approach to investigate alpha decay in super-heavy nuclei. A distinctive feature of our work is the utilization of a newly developed nuclear potential incorporating surface deformation. By modifying the well-established Woods-Saxon potential with a Gaussian term, we introduce an innovative framework to explore the behavior of alpha decay in heavy nuclei. This surface deformation concept extends our understanding of nuclear physics and enhances the precision of our calculations. In our study, we employ advanced techniques such as the Bohr-Sommerfeld conditions and Langer modifications to formulate the effective potential characterizing the interaction between the daughter nucleus and the alpha particle. These methodological enhancements uphold our ability to numerically determine the half-lives of the selected nuclei. Our research shows the applicability of these techniques in the domain of alpha decay studies. The primary outcome of our investigation is the substantially improved agreement between the calculated alpha decay half-lives and experimental data. By employing the modified nuclear potential, we enhance the accuracy of our calculations. This achievement underlines the potential of our approach to serve as a more precise and reliable tool for modeling alpha decay processes, offering a valuable contribution to the field of nuclear physics. Additionally, the insights gained from our study can inform further investigations into the behavior of super-heavy nuclei and their significance in astrophysical processes. In conclusion, this study demonstrates the novel application of a modified nuclear potential with surface deformation to improve the accuracy of alpha decay half-life calculations.

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