



The Fifth Eurasian Conference on
Nuclear Science and its Application
14-17 October 2008, Ankara - TURKEY



Determination of the Geometry of Some even-even Zirconium Isotopes

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THE AIM

- The aim is to investigate the **Zirconium nuclei** at the $A \sim 100$ region,
 - Predict properties of these nuclei,
 - Calculate energy spectra and $B(E2)$ values,
 - Determine the geometrical types of them.
- The goal is to apply the interacting boson model in its standard formulation IBM-1.



IN THIS STUDY

- We have investigated even–even nuclei in the $A \sim 100$ mass region within IBM-1 model.
- These isotopes with neutron numbers $N > 50$:
- Parameters are fitted to experiment* leading to
 - a description of 301 collective levels,
 - in 30 nuclei with 119 keV Root Mean Square deviation.

*National Nuclear Data Center, <http://www.nndc.bnl.gov>



IN THIS STUDY

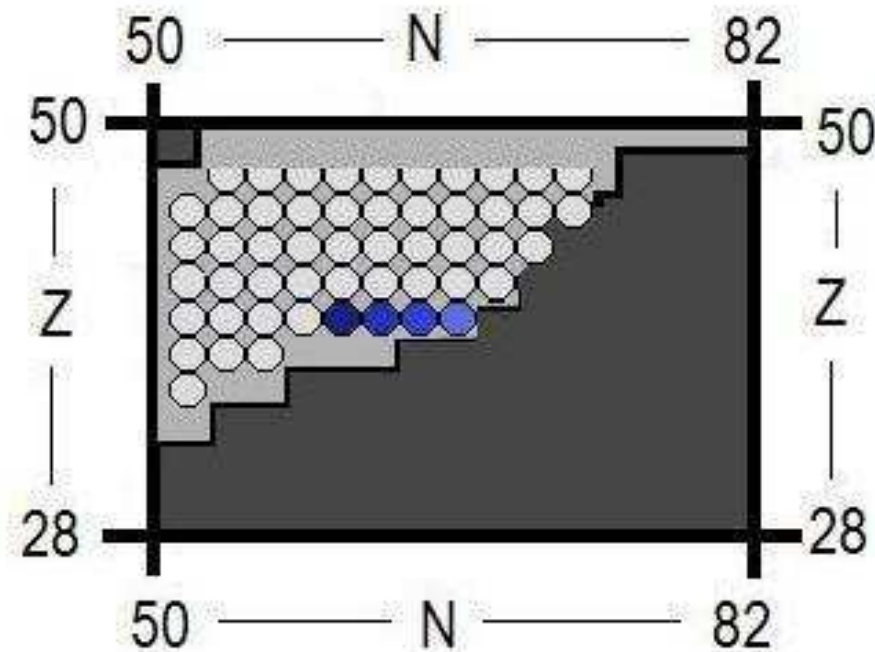
- The parametrization established on the basis of known elements to **unknown neutron-rich isotope**;

$_{-100}\text{Zr}$,

$_{-102}\text{Zr}$,

$_{-104}\text{Zr}$,

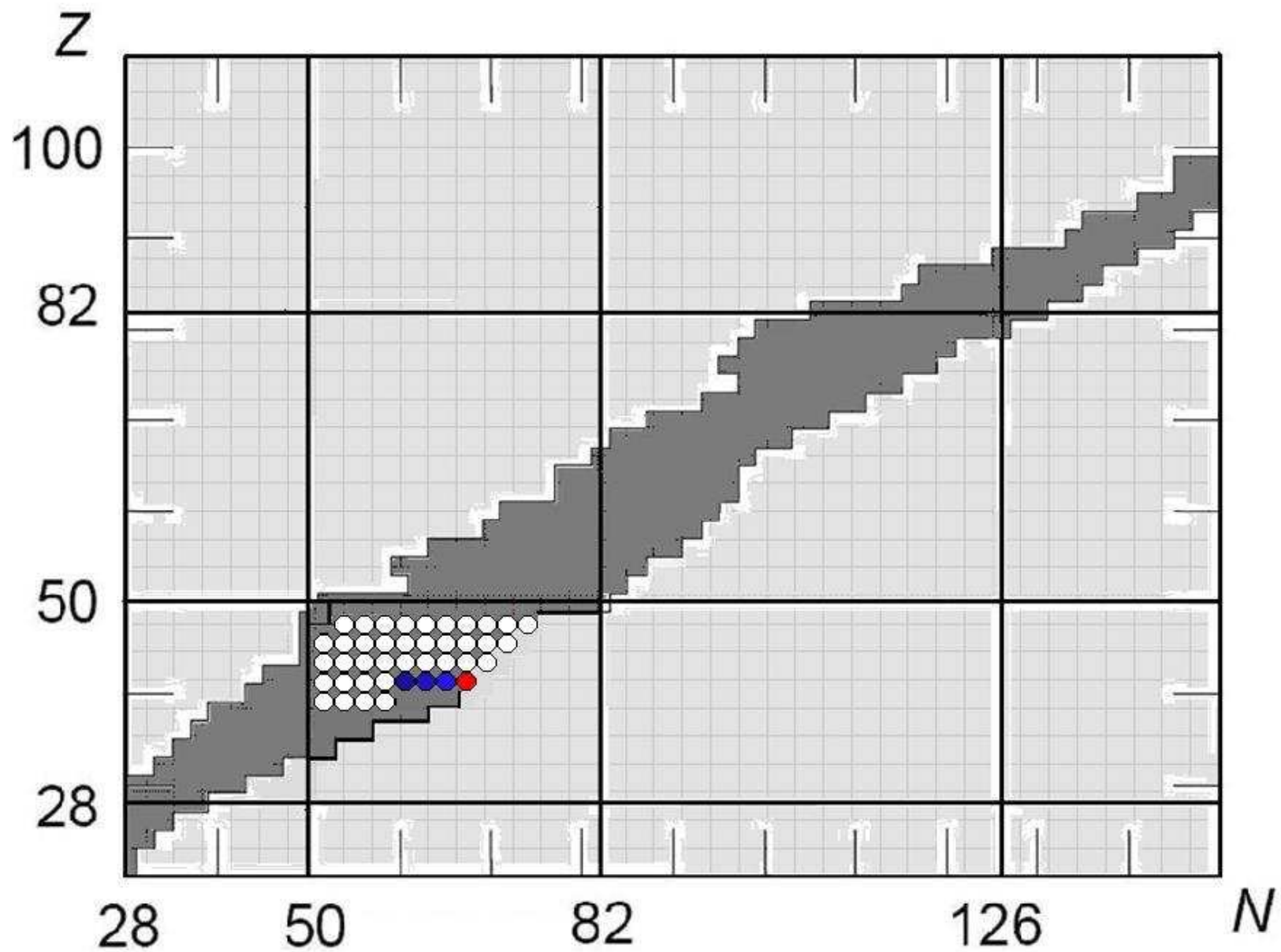
$_{-106}\text{Zr}$.





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PLAN

- IBM-1: Hamiltonian and parameters,
- $V(\beta, \gamma)$: Potential Energy Surface (PES),
- Results and discussion:
 - Energy spectra
 - Potential Energy Surface
 - $B(E2)$ transitions
- Conclusions



INTERACTING BOSON MODEL*

- The **IBM** is a semi-microscopic theory for nuclei, positioned intermediately between single-particle and collective models.
- The model contains a vibrational and a rotational limits.
- In the original version of IBM model, applicable to even-even nuclei, the basic building blocks are s and d bosons, with angular momenta $L=0$ and $L=2$, respectively.
- The s and d bosons can be interpreted as correlated or Cooper pairs formed by two nucleons in the valence shell.

*F. Iachello, A. Arima, The Interacting Boson Model, Cambridge Univ. Press, 1987.



INTERACTING BOSON MODEL

- The IBM-1 Hamiltonian

$$\hat{H} = \epsilon_d \hat{n}_d + \kappa \hat{Q} \cdot \hat{Q} + \kappa' \hat{L} \cdot \hat{L} + \kappa'' \hat{P} \cdot \hat{P} + q_3 \hat{T}_3 \cdot \hat{T}_3 + q_4 \hat{T}_4 \cdot \hat{T}_4$$

- E2 transition operator;

$$\hat{T}(E2) = e_b \hat{Q}$$

- Root Mean Square deviation.

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (E_i^{exp} - E_i^{cal})^2}{N}}$$



POTENTIAL ENERGY SURFACE

- Many nuclei find an interpretation in terms of the geometric model of Bohr and Mottelson*, where the vibrations are associated with oscillations of the nuclear surface.
- The geometric character of the nuclei can be visualized by plotting the potential energy surface $V(\beta, \gamma)$ obtained from the IBM-1 Hamiltonian in the classical limit**.

*A. Bohr, B.R. Mottelson, Nuclear Structure. II Nuclear Deformations, Benjamin, Reading, MA, 1975.

**J.N. Ginocchio, M.W. Kirson, Phys. Rev. Lett. 44 (1980) 1744,
A.E.L. Dieperink, O. Scholten, F. Iachello, Phys. Rev. Lett. 44 (1980) 1747.



POTENTIAL ENERGY SURFACE

$$V(\beta, \gamma) = \epsilon_d \frac{N\beta^2}{1+\beta^2} + \kappa \left[\frac{N}{1+\beta^2} (5 + (1+\chi^2)\beta^2) + \frac{N(N-1)}{(1+\beta^2)^2} \left(\frac{2}{7} \chi^2 \beta^4 - 4\sqrt{\frac{2}{7}} \chi \beta^3 \cos 3\gamma + 4\beta^2 \right) \right] \\ + \kappa' \frac{6N\beta^2}{1+\beta^2} + \kappa'' \frac{N(N-1)}{4} \left[\frac{1-\beta^2}{1+\beta^2} \right] + q_3 \frac{7N}{5} \left[\frac{\beta^2}{1+\beta^2} \right] + q_4 \left[\frac{9N}{5} \frac{\beta^2}{1+\beta^2} + \frac{18N(N-1)}{35} \frac{\beta^4}{(1+\beta^2)^2} \right]$$

$V(\beta, \gamma)$ depends on two quadrupole deformations**;

- β is the distance of a point from the origin and measures the total deformation,
- γ is the angle with the prolate axis and indicates the deviations from axial symmetry.

*P. Van Isacker, J.-Q. Chen, Phys. Rev. C 24 (1981) 684.

**B. Sorgunlu, P. Van Isacker, Nuclear Physics A (2008), doi:10.1016



RESULT AND DISCUSSION

- The parameters of the Hamiltonian were fitted to experiment* leading to
 - a description of 301 collective levels,
 - in 30 nuclei with 119 keV Root Mean Square deviation.
- After this procedure we calculate 24 collective energy levels and $B(E2)$ values for Zirconium isotopes from known to unknown region.

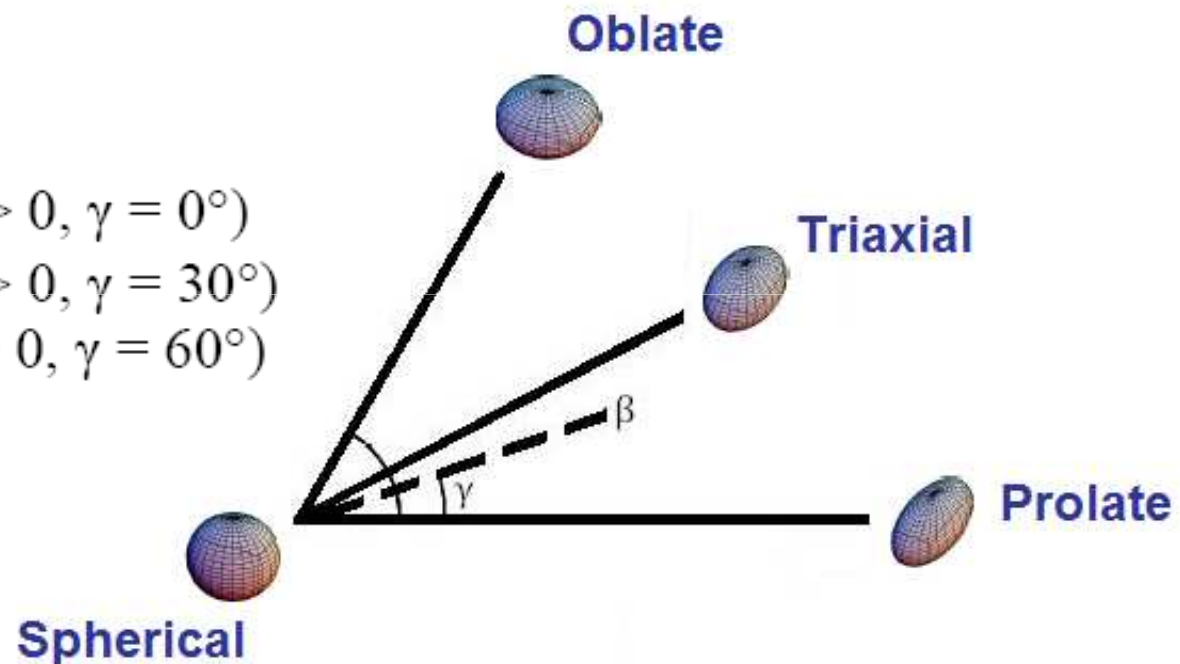
*National Nuclear Data Center, <http://www.nndc.bnl.gov>



RESULT AND DISCUSSION

- After fitted, Potential energy surfaces were plotted in the β, γ plane.

- **Spherical** ($\beta = 0$)
- **Prolate deformed** ($\beta > 0, \gamma = 0^\circ$)
- **Triaxial deformed** ($\beta > 0, \gamma = 30^\circ$)
- **Oblate deformed** ($\beta > 0, \gamma = 60^\circ$)



- β is larger in the IBM than in the geometric BM model*

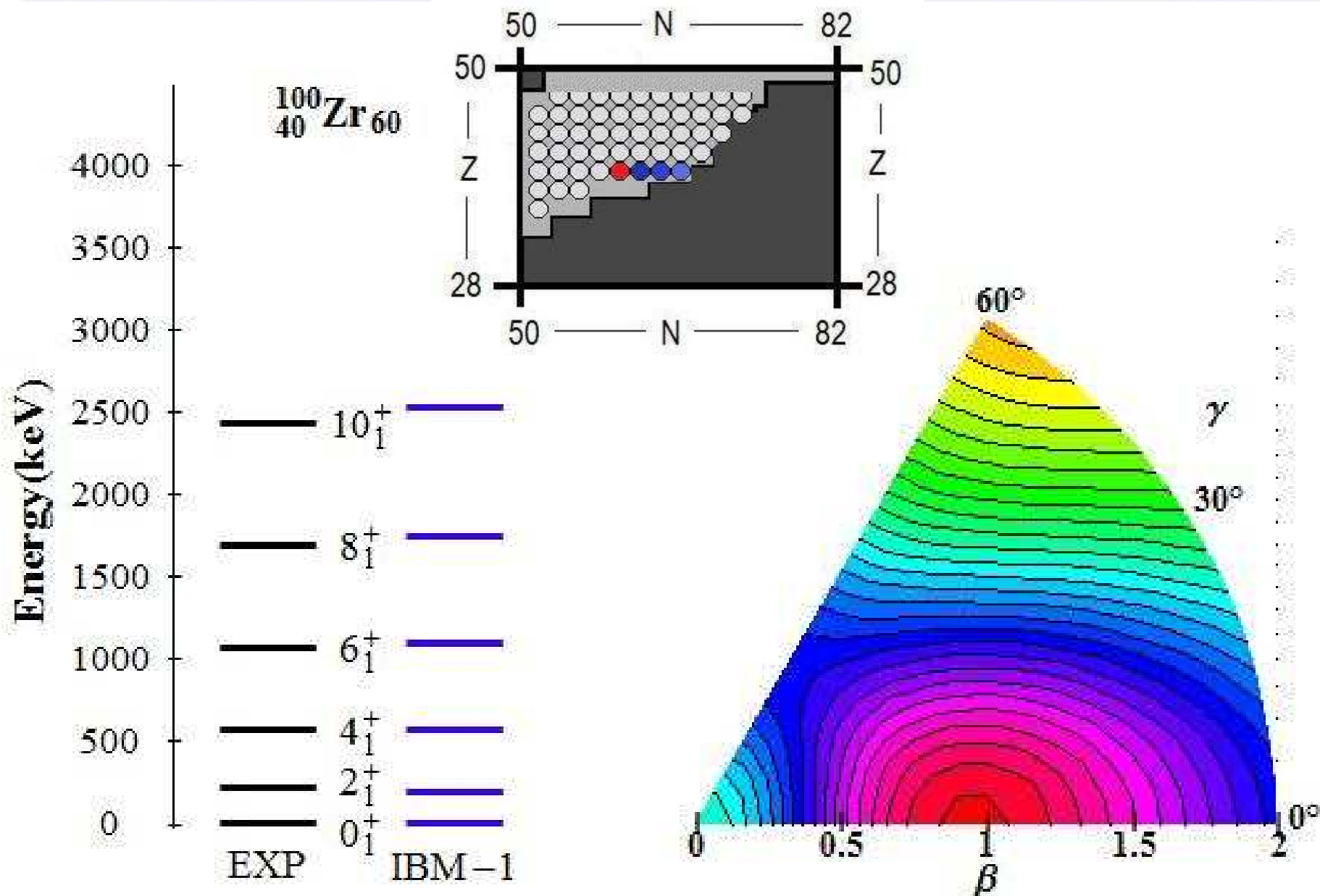
$$\beta_{\text{BM}} \cong 1.18 \frac{2N}{A} \beta_{\text{IBM}}$$

*J.N. Ginocchio, M.W. Kirson, Nucl. Phys. A 350 (1980) 31.



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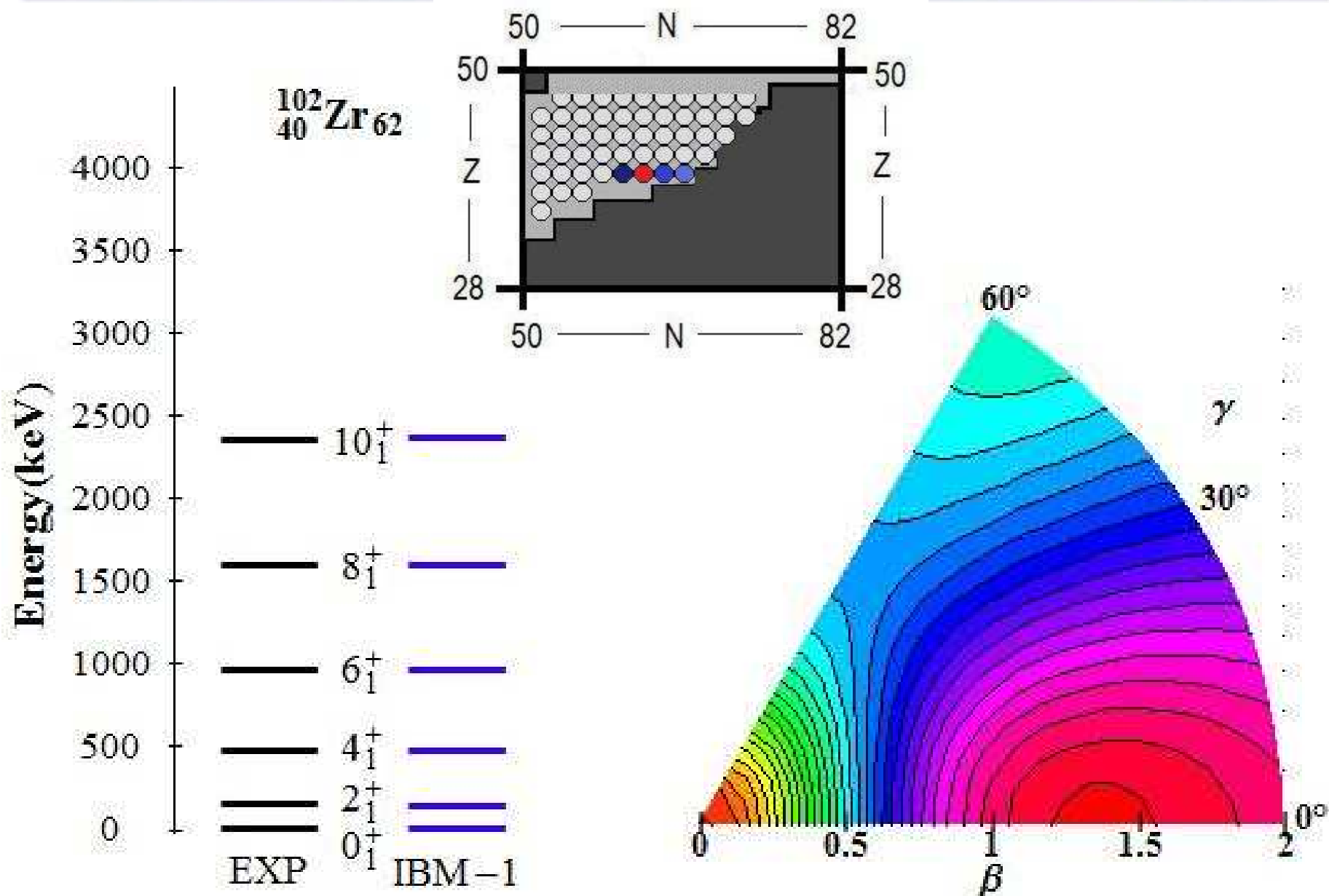
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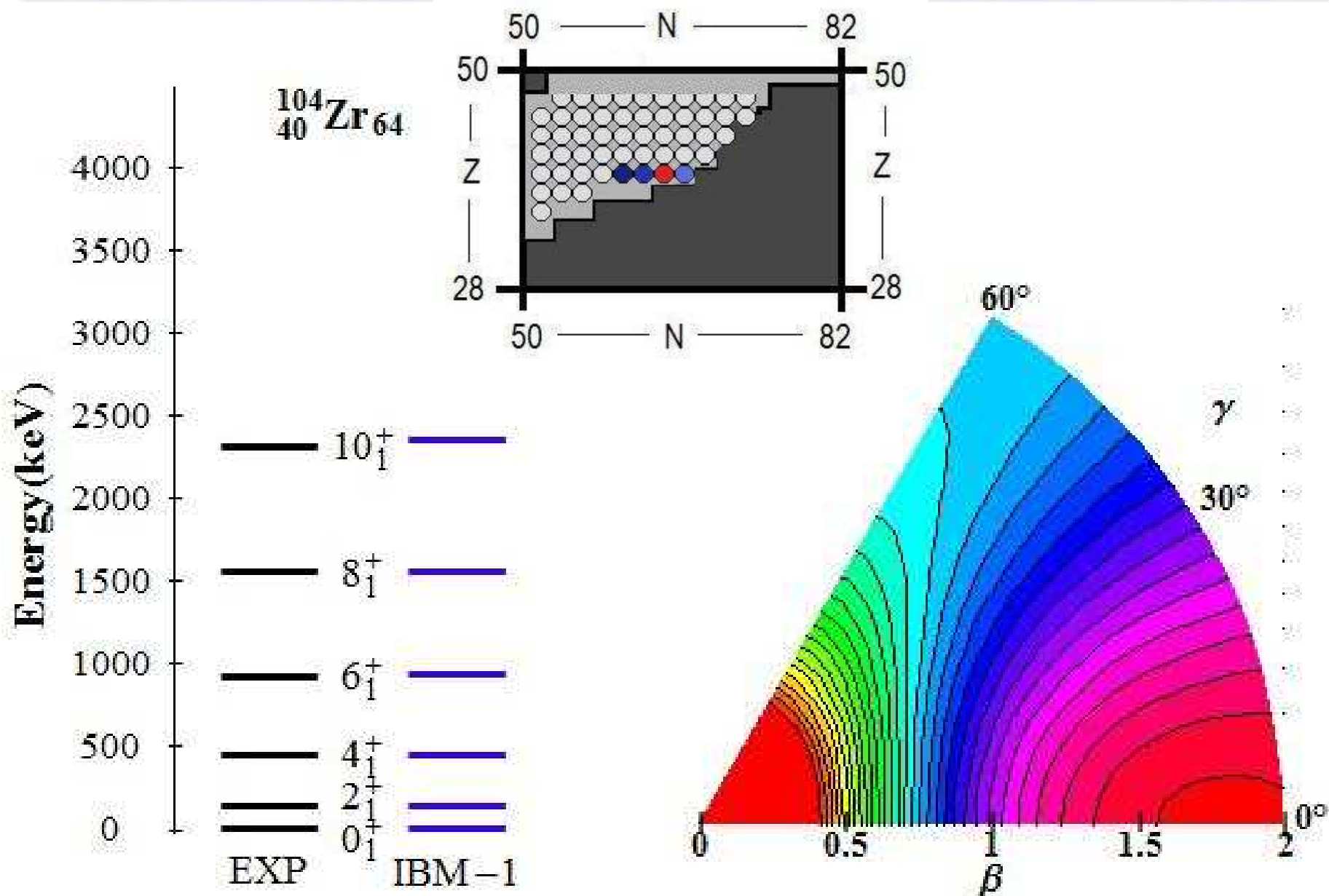
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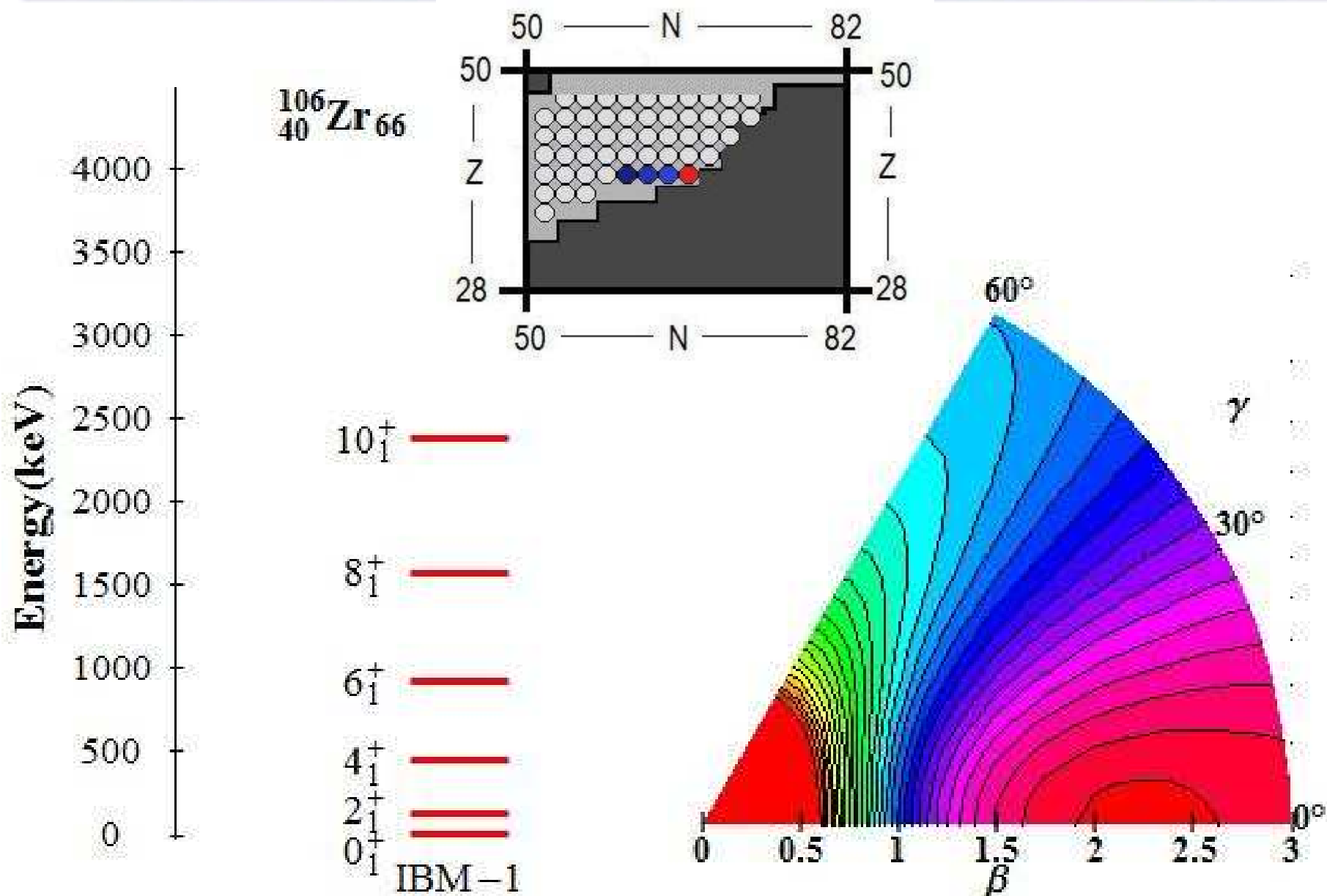
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$B(E2: 2_1^+ \rightarrow 0_1^+)$
($e^2 b^2$)

0.550
0.500
0.450
0.400
0.350
0.300
0.250
0.200
0.150
0.100
0.050

52

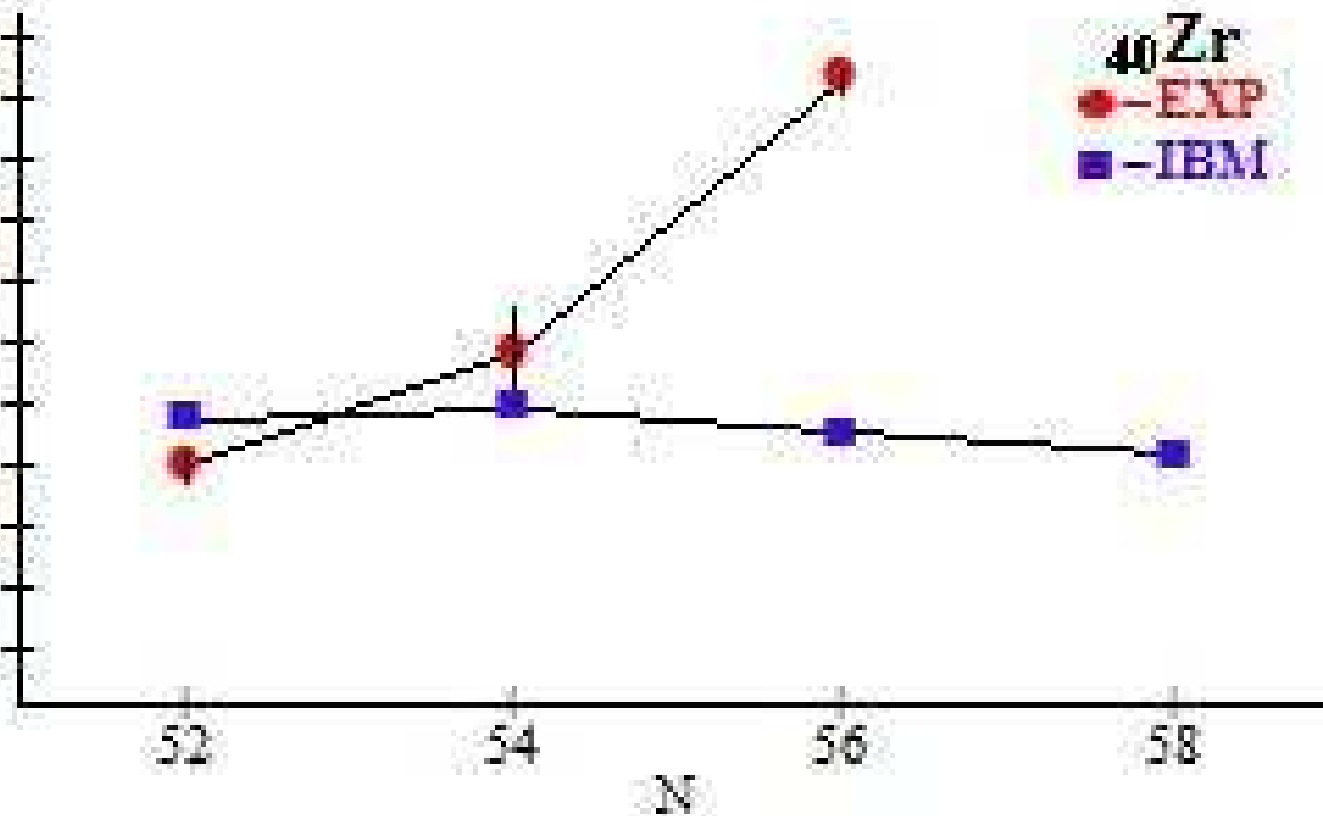
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56

58

N

^{40}Zr
● - EXP
■ - IBM





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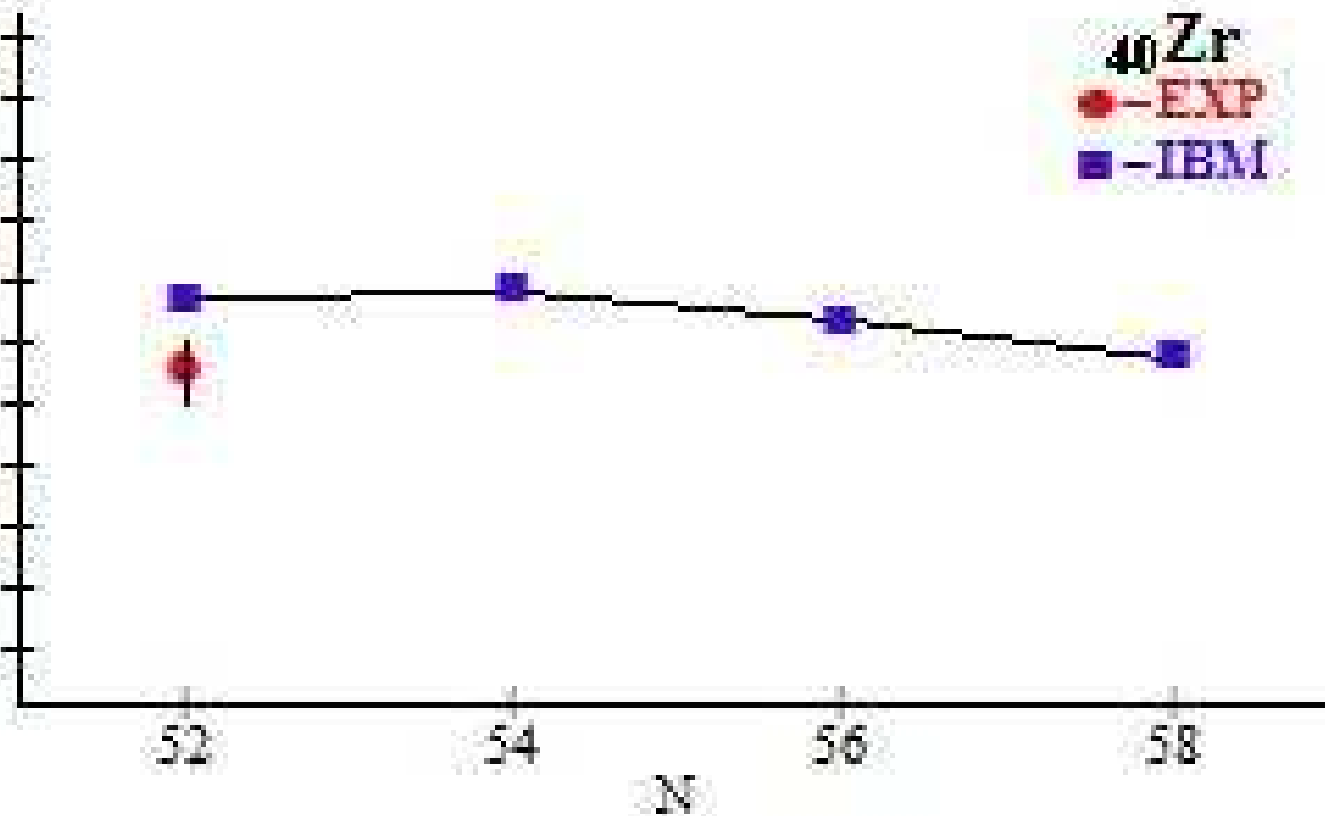
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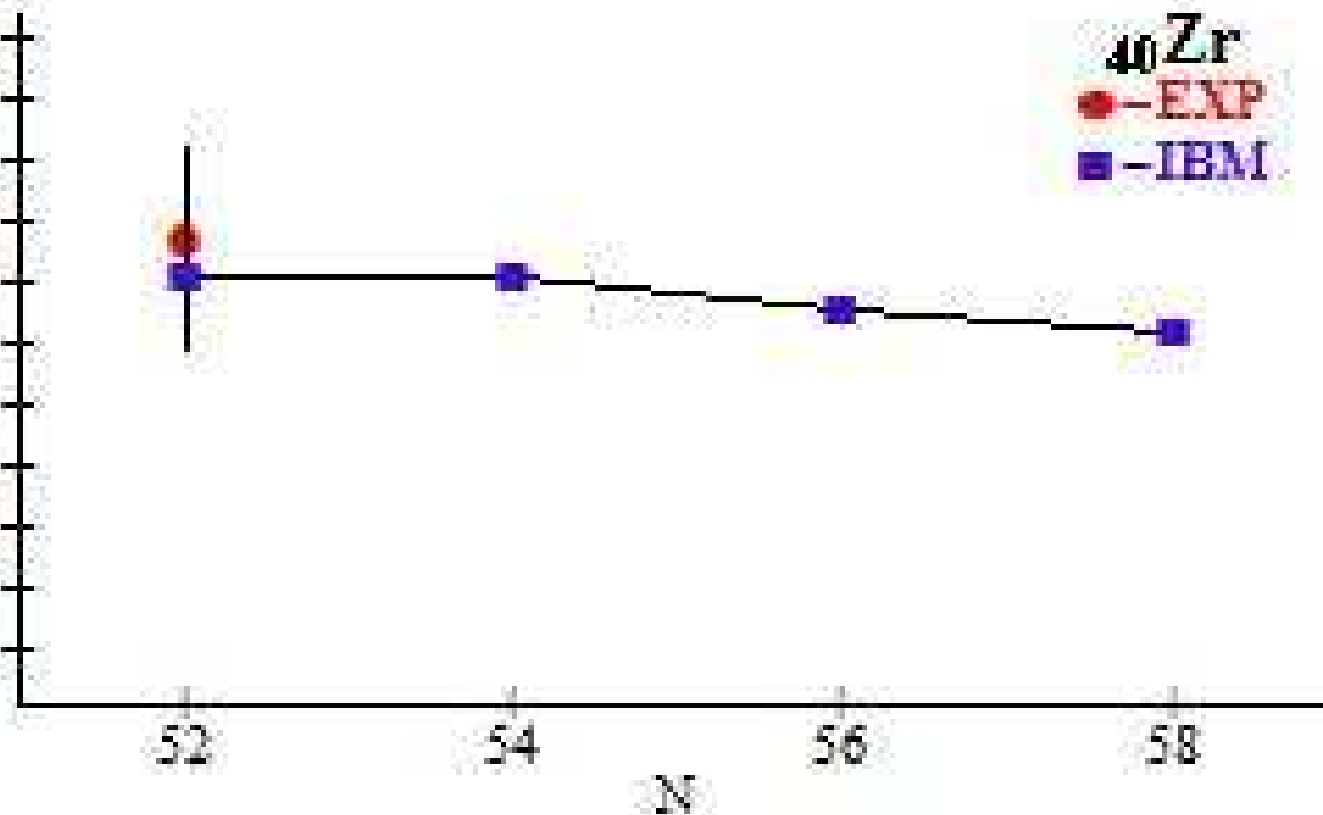
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^{40}Zr

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CONCLUSION

- Parametrizations have been done for Zr nuclei with general IBM-1 Hamiltonian.
- Collective levels have been calculated by using same parametrization.
- The comparison of the experimental and calculated levels is presented in Figures. There are a good agreement for all isotopes .
- $B(E2)$ values have been calculated and compared with experimental.



CONCLUSION

According to the potential energy
surfaces in the β - γ plane,
geometrical types of these nuclei
in $A \sim 100$ region are

PROLATE



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THANK YOU

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