SOME ELECTROMAGNETIC TRANSITION PROPERTIES AND MIXING RATIOS OF TELLURIUM ISOTOPES

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ABSTRACT

IBM-2 calculation is presented for the low-lying states in the even-even $^{122-128}$Te core nucleus. We developed the projection by using the F-spin formalism from the operator of IBA-2 model over the operator of IBA-1 model space. With the help of this projection IBA-2 Hamitonian parameters are obtained and we explore the energy levels and the electric quadrupole transition probabilities $B(E2;I_i \rightarrow I_f)$ and γ-ray E2/M1 mixing ratios for selected transitions. It was found that the calculated positive parity low spin state energy spectra of the even-even $^{122-128}$Te isotopes agree quite well with the experimental data.

Keywords: Interacting boson model, the electric quadrupole transition probability, mixing ratios, Low spin states
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1. INTRODUCTION

In recent years many works have been done on the structure of tellurium nucleus in recent years; V. Lopac *et al.* [1] studied on semi-microscopic description of even tellurium isotopes, E. Degrieck *et al.* [2] determined structure and electromagnetic properties of the doubly even Te isotopes , A. Subber *et al.*[3] apply the dynamic deformation model to the tellurium isotopes, M. Sambataro *et al.* [4] calculated the some of electromagnetic properties of Te and Cd isotopes with the framework of the interacting boson approximation, J. Rikovska *et al.*[5] studied dynamical symmetries in even-even Te nuclides, H. R. Yazar *et al.* [6] explore the energy levels and the electric quadrupole transition probabilities $B(E2;I_i \rightarrow I_f)$ and γ-ray E2/M1 mixing ratios for selected transitions of some even-even erbium isotopes with the help of this projection.

The interacting boson approximation represents a significant step forward in our understanding of nuclear structure. It offers a simple Hamiltonian, capable of describing collective nuclear properties across a wide range of nuclei, and is founded on rather general algebraic group theoretical techniques which have also found recent application to problems in atomic, molecular, and high-energy physics. The application of this model to deformed nuclei is currently a subject of considerable interest and controversy.

In the first version, IBA-1, no distinction is made between neutron and proton degrees of freedom. An unsatisfactory aspect of this model is that there is no clear connection with a microscopic structure of nucleus. The microscopic theory strongly suggests that it is important to treat the neutron and proton degrees of freedom independently. This has led to the introduction of the second, generalized, version of the IBA-model, IBA-2. In the second version, the neutron and proton degrees of freedom are treated explicitly. In this model the nucleus is described explicitly in terms of neutron (s_ν,d_ν) and proton (s_π,d_π) bosons. From calculations in the IBA-2 model it appears that the lowest levels are symmetric under the interchange of neutrons and protons. This symmetry is most easly discussed in terms of a variable called F-spins [7]. In the case of boson systems F-spin plays a role similar to that of isospin in the case of fermion systems.

The relation between the IBA-1 and IBA-2 model is obtained by identifying states of the former to the fully sysmetric i.e. maximal F-spin states of the latter model. Since the space of the IBA-1 model can be regarded as a subspace of the IBA-2 model there is a unique way to "Project" the operators of the IBA-2 model onto those of IBA-1. This projection can be carried out by using the F-spin formalism [8,9,10].

From these considerations it follows that IBA-1 and IBA-2 model can be related to each others and the states of the IBA-1 model can be identified with the fully symmetric states in the IBA-2 model. It is the purpose of this work to study these relations and applied to $^{122-128}$Te even mass isotopes.

The Project approximation used in this study has been extensively described by O. Scholten and H.R. Yazar for the neodynim, samarium, erbium and gadolinium isotopes [6,11,12]. We shall present here only the results of calculation and refer the reader to that work of the Project approximation for details [11,12,6]. In section 2 we study the positive parity spectra of the $^{122-128}$Te isotopes. In the same section E2 and M1 transition probabilities and electric quadrupole transition probabilities $B(\sigma L; I_i \rightarrow I_f)$ are analysed. Finally, the work is summarised in section 3.

2. THEORY AND METHOD OF CALCULATION

The IBM [13-15] provides a unified description of collective nuclear states in terms of a system of interacting bosons. The $^{122-128}$Te isotopes have 52 protons 70-76 neutrons, which fill the orbits below major shell closure at N=82, characterized by six neutron holes. Within the IBM, these structure or shape changes correspond to the system moving between the vibrational SU (5) and γ-unstable O (6) limits. The $^{122-128}$Te nucleus has been considered as a transitional nucleus from SU (5) to O (6) [15].

In the present work IBA-2 Hamiltonian parameters are normalized with the help of IBA-1 model hamiltonian. In the IBA-2 calculation the lowest states are indeed fully symetric, the calculation with the help of this projection gave good results for the excitation energies. Because of the admixtures of non-fully-symetric states in IBA-2 model space, the projection gave some difficulties and we had to renormalize the parameter (ε) and κ as shown in Figure 1.

In IBA-2 the neutron and proton degrees of freedom are treated explicitly. This has the advantage of being closer to a microscopic theory. The matrices that have to be diagonalized are,hovewer,much larger. One can regard the IBA-1 model space, in which neutron and proton degrees of freedom are not distinguished, as a subspace of the IBA-2 hamiltonian one can thus Project out its IBA-1 pieces [7, 12]. In the present work the relevant terms in the IBA-2 hamiltonian

$$H = \varepsilon\left(n_{d_\nu} + n_{d_\pi}\right) + \kappa\left(Q_{\nu,\pi}\right) + V_{\nu\nu} + V_{\pi\pi} \qquad (1)$$

Where the dot denotes a scalar product. The first term represents the single-boson energies for proton and neutron bosons and $n_{d\rho}$ is the number of d-bosons where ρ corresponds to π(proton) or ν(neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e the quadrupole-quadrupole interaction between neutron and proton bosons with strength κ. The quadrupol operator is

$$Q_\rho = [d_\rho^+ s_\rho + s_\rho^+ \tilde{d}_\rho]^{(2)} + \chi_\rho [d_\rho^+ \tilde{d}_\rho]^{(2)} \qquad (2)$$

Where ρ corresponds to π(proton) or ν(neutron) bosons and χ_ρ determines the structure of the quadrupol operator and is determined empirically. Where χ_ρ determines the structure of the quadrupol operator and is determined empirically. The square brackets in (2) denote angular momentum coupling.

The terms $V_{\pi\pi}$ and $V_{\nu\nu}$, correspond to interactions between like-bosons. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^\rho \left(\left[d_\rho^+ d_\rho^+ \right]^{(L)} . \left[\tilde{d}_\rho \tilde{d}_\rho \right]^{(L)} \right) \qquad (3)$$

The isotopes $^{122-128}$Te have $N_\pi = 1$, and N_ν varies from 4 and 5, while the parameters κ, χ_ν, χ_π and ε were treated as free parameters and their values were estimated by fittting to the measured level energies. This procedure was made by sellecting the "traditional" values of the parameters and then allowing one parameter to vary while keeping the others constant until a best fit was obtained. This was carried out iteratively until an overall fit was achieved. The best fit values for the hamiltonian parameters are given in Table 1

The numerical diagonalization has been carried out by using the code [16]. The values of the main parameters of the Hamiltonian are given in Table 1. The calculated excitation energies for $^{122-128}$Te isotopes as well as the

experimental ones are compared in Figure 2(a)-(b)-(c)-(d). The general agreement between experiment and model is quite good.

A successful nuclear model must yield a good description not only of the energy spectrum of the nucleus but also of its electromagnetic properties. The most important electromagnetic features are the E2 transitions. The B(E2) values were calculated by using the E2 operator. The E2 transition operator must be a hermitian tensor of rank two and therefore the number of bosons must be conserved. Since, with these constraints the general E2 operator can be written as [15]

$$T_m(E2) = e_\pi Q_\pi + e_\nu Q_\nu \tag{4}$$

Where Q_ρ are the Q_π and Q_ν boson quadrupole operators and e_π and e_ν are the "effective charges" for the proton bosons and the neutron bosons. For simplicity the "effective charges" e_π and e_ν were taken as equal (e=0.23 *eb.*). The B(E2) strength for the E2 transitions is given by

$$B(E2; L_i \to L_f) = 1/(2L_i+1)^{1/2} \left| \langle L_f \| T(E2) \| L_i \rangle \right|^2 \tag{5}$$

Some calculated B(E2) values from the ground state band and B(E2) ratios are given in Table 2.

Since tellurium nucleus has a rather vibrational-like character, taking into account of the dynamic symmetry location of the even-even tellurium nuclei at the IBM phase triangle where their parameter sets are at the O(6)-SU(5) transition region and closer to O(6) γ-unstable character and we used the multiple expansion form of the Hamiltonian for our approximation. In order to find the value of the effective charge we have fitted the calculated absolute strengths B(E2) of the transitions within the ground state band to the experimental ones. The best agreement is obtained with the value $e_\pi = e_\nu = e = 0.23$ *eb*, as shown in Table 2. The B(E2) values depend quite sensitively on the wave functions, which suggest that the wave functions obtained in this work are reliable.

E2:Ml multipole mixing ratios; In the nucleus, an electromagnetic exchange connecting a state of spin I_1 to I_2 can carry an angular momentum L between $I_1 + I_2$ and $|I_1 - I_2|$. In the rotation- vibration model, pioneered by Bohr and Mottelson [14], the low-lying, even-parity states of even-even nuclei are ascribed to the collective quadrupole motion of the nucleus as a whole.
The M1-E2 mixing parameter δ is defined as

$$\delta = \pm\sqrt{\frac{T(E2)}{T(M1)}} = \pm\frac{\sqrt{3}}{10}\frac{w}{c}\sqrt{\frac{B(E2|I \to I')}{B(M1|I \to I')}} \tag{6}$$

Where the ± sign must be chosen depending on the relative sign of the reduced matrix element [17]. The electric quadrupole and magnetic dipole transition probabilities T(E2) and T(M1) are, respectively,

$$T(E2|I \to I') = \frac{4\pi}{75}\frac{1}{\hbar}\left(\frac{w}{c}\right)^5 B(E2|I \to I')$$

$$T(M1|I \to I') = \frac{16\pi}{9}\frac{1}{\hbar}\left(\frac{w}{c}\right)^3 B(M1|I \to I') \tag{7}$$

The δ-mixing ratios for some selected transitions in $^{122\text{-}128}$Te isotopes are calculated from the useful equations as above and with the help of B(EI) and B(MI) values which are obtained from NPBEM(computer code which is subroutine of NPBOS package program)[16]; the results are given in Table 3. In general, the calculated electromagnetic properties of the tellerium isotopes do not differ significantly from those calculated in experimental and previous theoretical work [18,19].

3. SUMMARY AND CONCLUSION

In this paper we have carried out an analysis for the even mass tellurium isotopes based on the IBM-2. The boson core parameters have been obtained and the main results for energy levels and quadrupole transition probabilities agree very well with experiment. In general, good agreement was obtained when compared with experiment. The boson-boson interaction parameters were fixed by the calculations on the boson core nuclei. The results indicate that the energy spectra of all different quasibands of the even-even Te isotopes can be reproduced quite well. It is noticed, however, that the results of B(E2) calculations for even-even tellurium nuclei were in better agreement with the existing experimental data. The best fit values for the Hamiltonian parameters for even-even tellurium isotopes are given in Table 1 and the calculated energy values which are compared with the experimental data [19] are given in Figs. 2(a)-(b)-(c)-(d) for $^{122-128}$Te isotopes. The agreement is good for member of ground state, γ and β bands. The calculated values in this study show that the transitions connect the levels with the same parity and the E2 transitions are predominant.

A sensitive test of our projection is provided by comparing calculated B (E2) values with experimental predictions. The agreement between the values obtained in this analysis and the experimental results is good for ground state band and we hope that if the other parameters are normalized by means of this projection it can be considerably improved for γ-band and β-band for further work.

In this work, we have also examined the mixing ratio $\delta(E2/M1)$ of transitions linking the γ and ground state bands. The transitions which link low spin states and were obtained in the present work are in good agreement and show a little bit irregularities.

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Table 1. IBM-2 parameters; All parameters in MeV except χ_ν, χ_π

	ε	κ	χ_ν	χ_π	$C_{\pi\nu L}(0,2,4)$
^{122}Te	0.5033	0.008	0.01	0.002	0.002 0.003 0.002
^{124}Te	0.5128	0.010	0.01	0.002	0.002 0.003 0.002
^{126}Te	0.6198	0.020	0.02	0.003	0.002 0.003 0.002
^{128}Te	0.6210	0.022	0.02	0.003	0.002 0.003 0.002

Table 2. B(E2; $I \rightarrow I-2$) values for ground state bands of $^{122\sim128}$Te isotopes.

N	B(E2) (in e^2b^2)						B(E2) ratios		
	$4_1 \rightarrow 2_1$			$2_1 \rightarrow 0_1$			$(4_1 \rightarrow 2_1) / (2_1 \rightarrow 0_1)$		
	Theory	Ref.[20]	Exp.[18,19]	Theory	Ref.[20]	Exp.[18,19]	Theory	Ref.[20]	Exp.[18,19]
70	0.18	0.20	0.19	0.12	0.13	0.132±0012	1.50	1.53	1.46
72	0.16	0.18	0.1630	0.11	0.11	0.1138±0.0015	1.45	1.63	1.43
74	0.16	0.12	0.159	0.09	0.07	0.094±0.004	1.77	1.71	1.68

76	0.12	0.12	-	0.06	0.07	0.076±0.006	2.0	1.71	-

Table 3. Calculated and experimental mixing ratios for $^{122\text{-}128}$Te isotopes.

Isotopes	**Spin Parity** $I_i^\pi \rightarrow I_f^\pi$	**Theory** $\lvert\delta(E2/M1)\rvert$	**Experiment** [19,22] **δ(E2/M1)**
^{122}Te	$2_2^+ \rightarrow 2_1^+$	1.52	-3.48
	$4_2^+ \rightarrow 4_1^+$	0.37	-0.57
	$2_3^+ \rightarrow 2_1^+$	0.94	----
	$2_3^+ \rightarrow 2_2^+$	1.38	-0.3<δ<0.0
	$4_3^+ \rightarrow 4_1^+$	1.27	$1.3^{+0.3}_{-0.4}$
^{124}Te	$2_2^+ \rightarrow 2_1^+$	3.58	-3.55, -3.40
	$4_2^+ \rightarrow 4_1^+$	0.23	-0.18 -0.26
	$2_3^+ \rightarrow 2_1^+$	1.47	$1.5^{+0.6}_{-0.3}$ 0.52
	$2_3^+ \rightarrow 2_2^+$	0.54	0.10 0.23
^{126}Te	$2_2^+ \rightarrow 2_1^+$	3.80	-4.8, $-4.25^{+0.15}_{-0.01}$
	$4_2^+ \rightarrow 4_1^+$	1.63	$0.09<\delta<1.8^{+0.7}_{-0.4}$
^{128}Te	$2_2^+ \rightarrow 2_1^+$	3.23	$4.6^{+1.6}_{-1.0}$
	$2_3^+ \rightarrow 2_1^+$	3.18	$4.2^{+2.0}_{-1.0}$
	$3_3^+ \rightarrow 4_1^+$	1.24	1.4
	$3_3^+ \rightarrow 2_2^+$	3.84	0.45, $4.2^{+2.5}_{-1.2}$

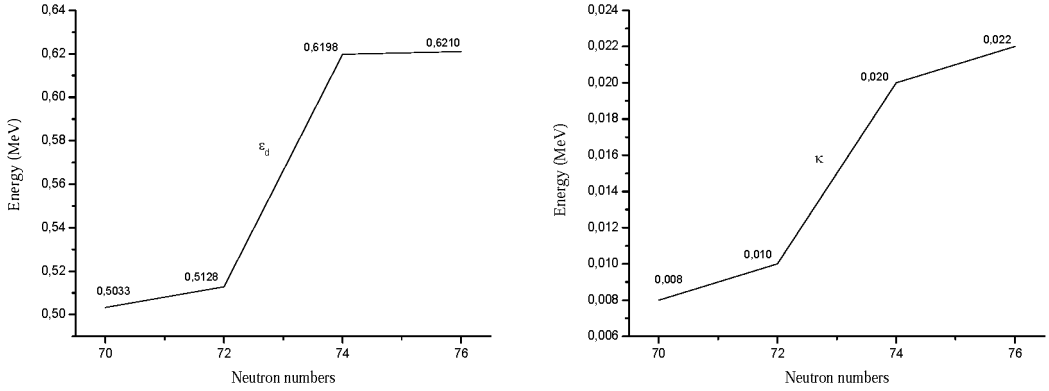
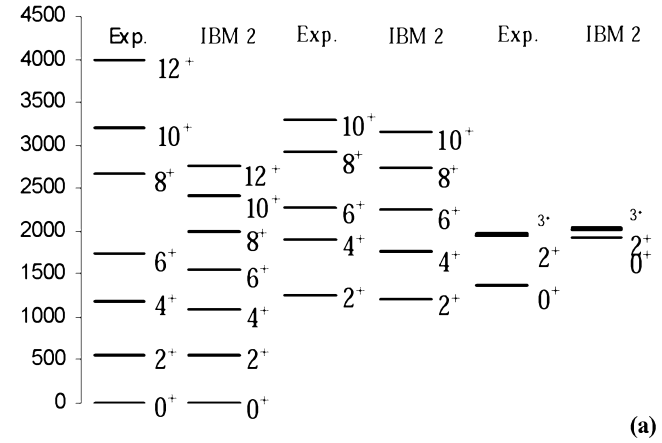
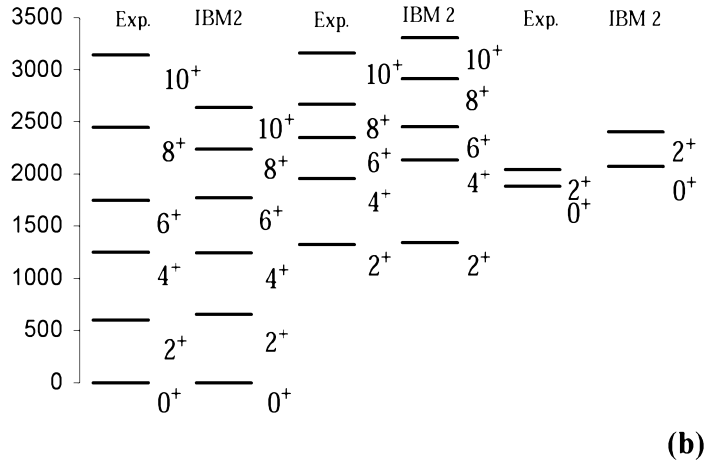


Figure 1. The parameters ε_d and κ employed for the IBA-2 calculations for Tellurium isotopes with even neutron numbers 70 up to 76.





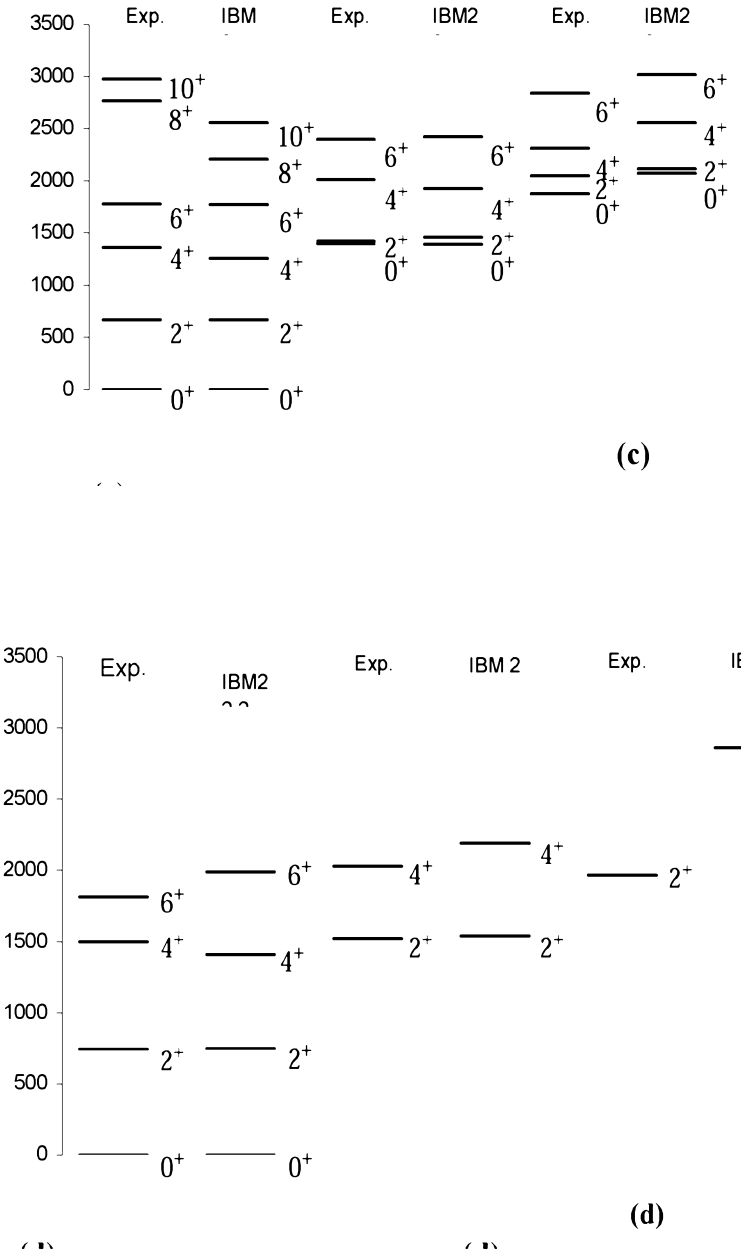


Figure 2. The three lowest rotational bands in spectra of (a) ^{122}Te and (b) ^{124}Te and (c) ^{126}Te and (d) ^{128}Te. In each band the experimental data are plotted on the left and calculated values on the right. (Energy in keV)

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