MOTION OF SPIN 1/2 PARTICLES IN A CURVED SPACETIME

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Lagrangian describing motion of non scalar particles in an external field includes additional terms containing spin variables. These terms change equation of motion of the particle due to interaction of spin with the external field [1]. Motion of particles with spin in curved spacetime was subject of studies of numerous authors [2,3]. It was shown in those works that equation of motion differs from geodesic equation due to a term, which is contraction the curvature with spin and velocity. However, there is still no general consensus on the behavior of particle with spin in external gravitation field. The goal of our studies is to develop a Lagrangian approach for spin 1/2 particles. We consider the particle as quasi-classical. In other words this means that motion of the particle is described by its coordinates $\{x^i\}$ and internal spin variables specifying spin degrees of freedom of the particle. Spin variables are elements of suitable spinor spaces which demand specifying an orthonormal frame along the worldline. Procedures of constructing a field of comoving frames along null-curve and timelike curve sufficiently differ from each other. In case of massive particle we could use length parameter s along the worldline, which specified timelike vector of unit length $\vec{n}_0=d\vec{x}/ds$. The vector defined 1+3 splitting of tangent space along the worldline used for specifying spinor variables [5]. In case of massless particle vector $\vec{n}=d\vec{x}/ds$ tangent to worldline of the particle has zero length. We need second null-vector, say $\vec{n}_+$, which will norm the vector $\vec{n}_-$ by the condition $<\vec{n}_-,\vec{n}_+>=1$. This determines 2+2 splitting of tangent space along the worldlines. It was shown in our work [7] that vector $\vec{n}_+$ specifies frequency of oscillations w of wave function of the particle, which defines energy of the particle $E=\hbar w$ referred to the frame. The spaces are constructed as follows. Pauli matrices $\{\sigma^\alpha\}$ referred to the rest-frame $\{\vec{n}_a\}$, where $\alpha=1,2,3$ or to the polarization frame $\alpha=1,2$ are introduced. The matrices generate local Clifford algebras which specifies two local spinor spaces attached to the worldline. These spaces are two spaces of spinor representations of the group SO(3) or SO(2) for massive and massless particle accordingly. The desired Lagrangian should depend on generalized coordinates $\{x^i\},\overline{\psi},\psi$ and their derivatives over s. At the same time the Lagrangian should contain covariant derivatives of spinor variables $\overline{\psi}$ and ψ. Moreover, the Lagrangian must contain term with square of coordinate velocity, which geodesic equation. Euler-Lagrange equations for spinor variables are expected to yield reduced form of Dirac equation. All these requirements determine the form of the Lagrangians spin 1/2 particles [5,6]. Euler-Lagrange equations are reduced to equations describing

motion of the spin along the particle worldline and the worldline shape. The equations for massive and massless spin 1/2 particles obtained this way become identical to Papapetrou equations for classic spinning particle [2], and for photon [7], accordingly.

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