INVESTIGATION OF $^{9}Be(p,\gamma)^{10}B$ REACTION

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INTRODUCTION

Reviews of estimated and experimental data on the interaction of charged particles with light nuclei [1, 2] highlight the importance of measuring the cross-sections of (p,γ) and (p,α) reactions, data that can further our understanding of light nuclei hydrogen and helium in the burn-cycle of stars. Also shown was the importance of modern theoretical methods in estimating cross-sections at astrophysical energies. Important information about nuclear structures, rates of nuclear reactions in the Sun and stars, and thus about nuclear fusion and heavy element abundance can be obtained by studying these reactions.

Hydrogen burn in second generation stars occurs via the proton-proton (pp) chain and CNO-cycle, with the $^{9}Be(p,\gamma)^{10}B$ reaction as an intermediate link between these cycles. The cross section of this reaction is well measured in the energy range 73 keV to 7.8 MeV [3]. However, estimation of the cross section in the range of astrophysical energies is encumbered by the presence of resonances. The difficulty can be alleviated by theoretical calculations. Our task is to generate a more accurate computation of the cross section of this reaction, its averaging over Maxwellian distribution (to determine the reaction rate); and to calculate the astrophysical S-factor and its extrapolation to zero energy.

The novelty of the introduced calculations is that they are carried out within the framework of a spectroscopic approach, since the basic characteristics of the radiative capture $^{9}Be(p,\gamma)^{10}B$ reaction are calculated with a wave function in the three-partial ααN-model. With the same wave function the cluster folding-potential and differential cross section of proton elastic scattering on ^{9}Be nuclei are calculated at several energy values. The differential cross-section obtained at E = 17 MeV is compared to the calculation of the cross-section computed within the framework of the optical model with the potential taken from [4]. From an analysis of the parameter energy dependence, their extrapolation to this energy is carried out. It is shown that both potentials correctly describe the experimental data.

METHODOLOGY

Knowledge of wave functions for input and output channels and spectroscopic factors for the disintegration of the ^{10}B nucleus in the ^{9}Be+p channel are needed for the calculation of $^{9}Be(p,\gamma)^{10}B$ reaction cross-sections, since the form of the electromagnetic transition Hamiltonian is well known.

In the two-body approach, the wave functions of input and output channels are generated in the optical interaction potential of a ^{9}Be+p system, parameters of which enters only once. The microscopic optical interaction potential is constructed within the framework of the cluster folding-model [5]. In this approach, the target is considered a three-particle system. The convolution is carried out over cluster density and pair interclusters of αn and np interactions. Since mathematically this is a four-body problem and the calculation of matrix elements of the convolution is very complicated, we have limited the potentials to be split only by orbital momentum.

The experimental spectroscopic factors for joining a proton to a ^{9}Be nucleus, or for separation of a proton from a ^{10}B nucleus, were determined from reactions of proton transfer of ^{9}Be(d,n)^{10}B and ^{9}Be(^{3}He,d) ^{10}B [6]. However, the spectroscopic factors obtained from different reactions are not in agreement with each other. Therefore we have used in our calculation the value S = 0.532, taken from Bojarkina's work [7].

1. Calculation of cluster folding potential

The interaction potential of a proton with a ^{9}Be nucleus in the cluster folding-model is expressed as an integral:

$$V_{p-^{9}Be}(\vec{R}) = \left\langle \Psi_{^{9}Be}(\vec{x},\vec{y})\varphi_{p} \middle| V \middle| \Psi_{^{9}Be}(\vec{x},\vec{y})\varphi_{p} \right\rangle, \qquad (1)$$

where (x, y) is a set of Jacobi coordinates; R a radius-vector connecting mass-centre of the incident proton and the ^{9}Be nucleus. The potential in Formula (1) is a sum of three potentials:

$$V = V_1(r_{12}) + V_2(r_{13}) + V_3(r_{14}), \qquad (2)$$

where V_k are potentials of intercluster interaction between fragments i and j, depending on their mutual distance r_{ij}.

The wave function of the ^{9}Be nucleus was used in ααn-model [8]. It more completely describes ^{9}Be properties.

In the cluster folding potential, it is necessary to set the parameters of cluster-cluster potential. We have used two potentials: the nucleon-nucleon and nucleon-alpha particle potentials taken from work [5,8].

2. Test of the folding potential and its comparison with the phenomenological one

As a test of the obtained cluster-folding potential and its comparison to the phenomenological potential, an analysis of proton elastic scattering on ^{9}Be nucleus in the energy range from 13 to 180 MeV was carried out. The analysis of experimental data for proton elastic scattering on ^{9}Be nucleus [4] was made with use of a program known as ECIS88. In the first stage, all parameters were varied until they came to an agreement with the experiment. Then the average radius value of real, imaginary and spin-orbit parts were selected and fixed over the whole energy range. After that, a search for the optimal parameters of the optical potential was carried out. The results of calculations, with use of the obtained optical potentials, are shown in Figure 1a. Then the energy dependence of the optical parameters was investigated. The obtained cluster-folding potential was parameterized in Wood-Saxon form with parameters V_o = 85.0 MeV, R_o = 0.85 fm, and a = 0.953 fm. The parameters were applied to get a description of proton scattering at energy E_p = 17 MeV. The results of the calculations are shown in Fig. 1b. Up to an angle of 80°, qualities of data description with the cluster-folding potential are comparable with the phenomenological description. Thus, it can be used in further calculations of radiative capture cross-sections.

3. Calculation of radiation capture cross-section

There are two well-determined resonances in the cross-section of $^9Be(p,\gamma)^{10}B$ radiative capture reaction. Therefore, the direct capture model can not be used, for any model needs to take into account a coupling of scattering state in the entrance channel with the resonance levels. However, an attempt to describe the cross-section with the aid of the Breit-Wigner standard formulas turns out to be impossible. These formulas do not take into consideration the dependence of the resonance level parameters on energy. For a correct account of such dependence, one needs to proceed from the more exact quantum-mechanic theory of resonance reactions.

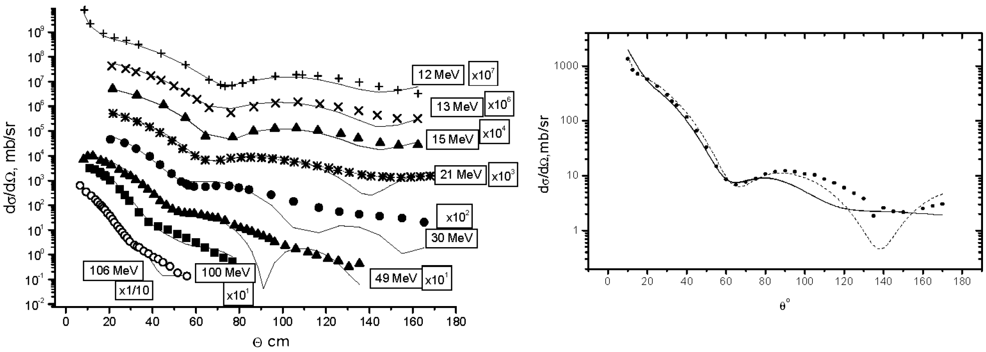

Figure 1 Differential cross sections of proton elastic scattering on ^{9}Be. In (a), the range of energies is from 13 to 160 MeV; dots, diamonds and triangles show experimental data from [8]; while solid curves show calculation with the optical potential; In (b) the energy is 17 MeV; dots are experimental data from [8]; the solid curve shows a calculation with the folding potential; the dash curve shows a calculation with the optical potential where parameters are obtained by extrapolation from energy of 15 MeV.

In our approach, the wave function at the entrance channel has the following form:

$$\Psi_i = \psi_1(\vec{x})\psi_2\chi(\vec{y}) + \Lambda_{res}\Psi_{res}(\vec{x},\vec{y}), \quad (3)$$

where

$$\Lambda_{res} = \frac{1}{E - E_{res} - \Delta_{res} + i\Gamma_{res}/2}\left\langle \chi(\vec{y})\psi_1(\vec{x})\psi_2 \middle| V_{12} \middle| \Psi_{res}(\vec{x},\vec{y})\right\rangle, \quad (4)$$

$$\Gamma_n = 2\pi\int \langle 1|V_{12}|1\rangle\langle 1|V_{12}|1\rangle \rho(E,\Omega)d\Omega$$

$$\Delta_n = \mathrm{P}\int \frac{\langle 1|H|1\rangle\langle 1|H|1\rangle}{E-E'}\rho(E',\Omega)dE'd\Omega .$$

Substituting (4) into the expression for the cross section, we obtain:

$$\frac{d\sigma}{d\Omega} = \frac{v_i}{v_{in}}\frac{2\pi}{\hbar}\left[\langle f|W_-|i\rangle + \frac{\langle f|W_-|res\rangle\langle res|V|i\rangle}{E-E_{res}-\Delta - i\frac{\Gamma}{2}}\right]^2 \frac{V_f}{(2\pi)^3}\frac{q^2}{\hbar c} . \quad (5)$$

The differential cross-section of the $^{9}Be(p,\gamma)^{10}B$ reaction is calculated by Formula (5). From the calculation, it is suggested that either E1 or M1 transitions give contribution and that the resonance is at energy 989 keV.

4. Calculation of the astrophysical S-factor

In stars and thermonuclear reactors, reactions take place on the energy order of 10 keV. Therefore, as Salpeter has remarked, the experimental cross sections, having the lowest limit of 50 keV, need to be extrapolated into the astrophysical region. However, the cross section itself is not convenient for extrapolation by reason of its irregular behavior at small energies. It is more convenient to extrapolate the S-factor, which is connected with the cross section by the formula

$$S = \frac{\sigma E_{\text{ö.ì.}}}{P}. \tag{6}$$

Here, P is the penetrability of the potential barrier. As energy decreases it rapidly decreases, since it behaves as the cross section. Such behavior stipulates the smoothed dependence of the S-factor on energy, giving accuracy to its extrapolation. Our calculations were carried out with use of the Gamov penetrability.

According to Salpeter, the S-factor is a magnitude which slowly changes as energy varies and at small energies it tends to be a constant. We quadratically extrapolate the S-factor by

$$S(E) = S(0) + AE + BE^2. \tag{7}$$

Integrating over the energy of the differential cross section (Formula 5), we get the total cross-section of radiation capture. Calculating the Gamov penetrability, we computed the S-factor in the region of energies from 20 keV to 1.8 MeV. The obtained values are shown in Fig. 2a as crosses. Quadratically extrapolating this data, we get the following values of parameters:

$S(0) = (0.226 \pm 0.001)$ keV·b; $A = (-0{,}0002 \pm 0{,}00001)$ b; $B = (-2{,}9 \pm 0{,}5)10^{-7}$ keV^{-1}·b

The obtained S-factor locates closely to the curve in [9]. In this paper, the value of the S-factor at zero is S(0) = 0.21 keV·b.

5. Calculation of the averaged reaction rate

We have calculated reaction rates of the $p+{}^{9}Be \rightarrow {}^{10}B+\gamma$ process by averaging over the Maxwell velocity distribution for the protons. This process occurs during stationary burning in the entrails of the Sun and stars. The calculations are based on theoretical values for reaction cross sections at different values of proton energy.

In the laboratory system, the reaction rate averaged over Maxwellian distribution was calculated as

$$\langle \sigma v \rangle = \left(\frac{8}{\pi}\right)^{1/2} \left(\frac{\mu}{\kappa T}\right)^{3/2} \frac{1}{m^2} \int_0^\infty e^{-\frac{\mu E}{m \kappa T}} \sigma(E) E dE. \tag{8}$$

Numerical integration is carried out from $E_{\min}$ to $E_{\max}$ and the integral's dependence on these limits is discussed below.

Figure 2b shows results of the averaged cross sections. The cross-sections obtained with the Gamov penetrability are shown as dots and their comparison with calculated values from earlier work [10] is shown as a solid curve. Our calculation differs from that in work [10] by the region of integration over the energy in Formula (8). We integrated in the following regions of energies: $E_{min} = 0.02$ MeV, $E_{max} = 1.8$ MeV; in [10], $E_{min} = 0.06$ MeV, $E_{max} = 1.8$ MeV. As the figures show, the difference is small for large T_9 and becomes apparent at $T_9 < 0.01$. It is stipulated by the sharp exponential dependence of the distribution in the region of low energies 0.001-0.05 MeV and by the high sensitivity of the integral in this region.

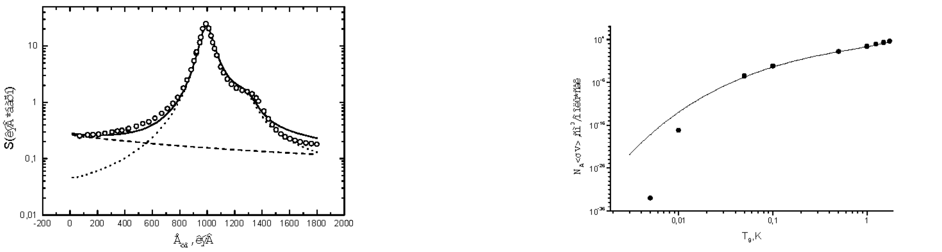

Figure 2. (a) shows the S-factor for the $^9Be(p,\gamma_0)^{10}B$ reaction. The dash curve is for the direct capture; the doted is for the resonance capture; and the solid line is their sum. The experimental data (circles) are from [10]; (b) shows reaction rates for $p+{}^9Be \rightarrow {}^{10}B+\gamma$ calculated with the Gamov penetrability and averaged over Maxwellian distribution. Dots denote our calculation, while the curve shows a computation from [10].

CONCLUSIONS

The method of calculation of the radiation capture differential cross-section has allowed a more correct description of the available experimental data. In our computations we used the cluster-folding potential. Calculation of the averaged reaction rates showed their high sensitivity to the low-energy region of the Maxwellian distribution, that is, the sensitivity to the lowest integration limit. The calculated values of the astrophysical S-factor are in agreement with the results obtained in [9].

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