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WEIGHTED RESIDUAL APPROACH TO THE SOLUTION OF THERMALHYDRAULICS PROBLEMS

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ABSTRACT

The possibility of application of weighted residual approaches to determine approximate, but fast solutions of thermalhydraulics problems is presented. The method has been successfully applied in a separate study to model a LOCA analysis /1/.

ÖZET

Isıl-hidrolik sorunların yaklaşık, ancak hızlı çözümlerinin bulunmasına dönük olarak ağırlaştırılmış kalıntılar yaklaşımının uygulanma olanağı sunulmaktadır. Yöntem, bir soğutucu kaybı kazası çözümlemesine diğer bir çalışmada, başarıyla uygulanmıştır /1/.

INTRODUCTION

Whatever are the specificalities, the thermalhydraulics problems may be formulated as:

$$\frac{\partial U(\underline{r},t)}{\partial t} = H(\underline{r},t) \quad U(\underline{r},t) \; ; \qquad (1)$$

where U(r,t) is a time dependent column matrix whose elements are the time dependent system variables; H(t) is in general a complicated G x G square matrix operator, G being the number of the system's variables.

If an analytical solution to the above equation is not known, it is customary to attack it by the conventional finite difference approach, which can be considered in general good; but in many cases, like for instance in LOCA type of analysis, the numerical solution of the equation in question is enormously time

consuming. It is not thus rare that one hears of many hours of computation on even the fastest computers. Further, the mechanism of the production of the finite difference numerical solution is so complicated and still full of so many unknowns that, often it may even turn out to be difficult to believe in the health of this solution. If for instance, numerical oscillations occured throughout the output, it may not be trivial whether this comes from the inherent drawbacks of the proposed finite difference formulation or rather reflects a true prediction of the physics behind the problem in hand /6/.

THE WEIGHTED RESIDUAL APPROACH

An expansion of the finite difference method is the finite element method which happens to be gradually introduced for the solution of the thermalhydraulics problems /2/.

A more general method shall be the synthesis technique equipped with the weighted residual criteria. This technique embodies both the finite difference and finite element methods. It can also be understood as the generalization of the variational method, in the sense of freeing most of the restrictions related with this /3/.

Thus pick up before hand, intuitively a trial vector for U(r,t), that is:

$$\bar{U}(\underline{r},t) = \sum_{i=1}^{I} V_i(\underline{r},t)\, N_i(t) \qquad (2)$$

the vectors $V_i(\underline{r},t)$ are thus known and $N_i(t)$'s are scalar unknowns. The vectors $V_i(\underline{r},t)$ are composed of intuitively proposed matrix elements.

It should be noted that the expansion proposed via Eq. 2 is not the only form one can guess. For instance, prediction would have been made about the time dependency, and the space dependency would be taken as the unknown.

It is also worth to note that the choices of the trial vectors $V_i(\underline{r},t)$ of the above expansion are totally arbitrary. Besides, they are proposed, from the physics point of view, intuitively.

Now, with the expansion in question go back to the original equation. Since this is only an approximate

solution, the equality of Eq. 1 will be broken and one will obtain a time and space dependent residual vector:

$$R(\underline{r},t) = \frac{\partial \bar{U}(\underline{r},t)}{\partial t} - H(\underline{r},t)\ \bar{U}(\underline{r},t) \qquad (3)$$

Choose on the other hand, weighting vectors, $W_i(r,t)$, $i = 1,\ldots,I$, the dimension of these vectors being also G, the number of system's variables, in such a way that the chosen vectors reflect, as good as possible, the importance displayed by the error vector due to the use of the approximate vector $U(\underline{r},t)$, at the phase point (r,t).

Thus weight the right hand side of Eq. 3 with $W_i^T(r,t)$, where the symbol T refers to the matrix transposition operation. Integrate the product expression over a convenient space and force the new product expression to vanish. This way one reinsures in a weighted and integrated residual sense the broken equality of the original equation.

Repeating the operation in question I times with each of the chosen weighting vectors, one gets I scalar equations for the I scalar unknowns $N_i(t)$'s.

Thus

$$\int_{V_c} dV\ W_i^T(\underline{r},t) \sum_{j=1}^{J} \left\{ \frac{\partial |V_j(\underline{r},t)\ N_j(t)|}{\partial t} - H(\underline{r},t) V_j(\underline{r},t)\ N_j(t) \right\} = 0, \qquad i = 1, \ldots, I. \qquad (4)$$

V_c, here denotes the volume of the "convenient space" chosen for the integration in question. It could be the volume of a fuel rod, a coolant channel, etc.

Eq. 4 thus displays a set of I equations for the I unknows $N_j(t)$'s. Each of these equations is a first order differential equation with respect to the unknown $N_j(t)$.

The solution of the set of I equations for the I unknown $N_j(t)$'s shall be obtained in a much shorter computation time as compared to the computation time that would be required by the conventional space-time finite difference scheme.

It seems practically impossible to define a general error criteria for the proposed approximate method. What builds the health of the solution is, of course, the degree of precision carried out by the initial guesses at the level of Eq. 2, along with the appropriateness in the choice of weighting vectors and the

domain of integration at the level of Eq. 4.

The validity of the approximate predictions should thus be evaluated across the more exact solutions for a few cases. One can thus develop a suitable intuition upon which he can base the guesses related to the unknown vector $U(\underline{r},t)$, for his succeeding analyses.

CONCLUSION

The proposed method has already been successfully used to model a LOCA analysis /1/. Investigations are being still carried out to apply it to a very fast working modeling of the complicated occurances of the TMI accident /5/. An attempt is further made to check the health of the approximate predictions /7/. A detailed analysis that considers the effect on the final result, of various space dependencies and various weight vectors used in Eqs. 1-3 is being designed /8/. This analysis will hopefully show the best guess materials to be associated, in thermal hydraulics problems that will be studied, with the weighted residual criteria, the main mathematical tool of this presentation.

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