

FOLDING POTENTIALS FOR CALCULATIONS OF CROSS SECTION OF ${}^{6,7}\text{Li}(\text{d}, {}^{6,7}\text{Li}){}^2\text{H}$ AND ${}^6\text{Li}({}^3\text{He}, \text{d}){}^7\text{Be}$ REACTIONS

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1. Direct method of radiative capture reaction cross section determinations at low energy range encounter with significant difficulties. It is connected with sharp exponential decreasing of cross sections looking to zero of energy and come along increasing of gamma-ray background. For avoiding of these difficulties in present time it is built underground laboratories, or introduced background subtraction procedure. Using of thick targets distorts resonance peaks. Thus in last time widely used indirect methods of cross section evaluation: Coulomb's dissociation, Trojan horse method, transfer reaction method [1].

It is well-known that the radiative capture $\text{d}(\alpha, \gamma){}^6\text{Li}$ reaction is of great interest as one of sources of the ${}^6\text{Li}$ production in the Universe [2-3]. However, up to now in nuclear astrophysics the problem of obtaining reliable experimental astrophysical S -factors, for the $\text{d}(\alpha, \gamma){}^6\text{Li}$ reaction at astrophysically relevant energies E ($E < 300$ keV) is still unsolved. Direct measurement is very large experimental task, because cross section is formed by E2 transition, and at low energies it decrease rapidly.

Reaction of radiative capture of ${}^6\text{Li}(\text{p}, \gamma){}^7\text{Be}$ plays important role of ${}^7\text{Li}$ isotope formation, which is produced via electron capture by final ${}^7\text{Be}$ nucleus. Astrophysical S -factor values for this reaction is obtained at $E_p > 150$ keV with errors of 30% in low energy range [4]. Calculated reaction rates of this reaction for temperature $T_9 \leq 1.5$ became higher than recommend values from work [5]. This discrepancies can be connected with large experimental errors and ambiguity of extrapolation procedure of cross-sections and S -factors in low energy range. For astrophysical S -factor evaluation of ${}^6\text{Li}(\text{p}, \gamma){}^7\text{Be}$ reaction, it is proposed indirect method, using of proton transfer reaction ${}^6\text{Li}({}^3\text{He}, \text{d}){}^7\text{Be}$, which gives possibility of nuclear vertex constant determination for virtual decay of ${}^7\text{Be} \rightarrow {}^6\text{Li} + \text{p}$. For astrophysical S -factor evaluation of $\text{d}(\alpha, \gamma){}^6\text{Li}$ reaction will be used ${}^6\text{Li}(\text{d}, {}^6\text{Li}){}^2\text{H}$ reaction.

2. Let us consider Feynman's diagram of ${}^6\text{Li}(\text{p}, \gamma){}^7\text{Be}$ and ${}^6\text{Li}({}^3\text{He}, \text{d}){}^7\text{Be}$ reactions, which presented on figure 1. For description of both processes it is needed vertex function of ${}^7\text{Be} \rightarrow {}^6\text{Li} + \text{p}$ channel. For description of ${}^6\text{Li}({}^3\text{He}, \text{d}){}^7\text{Be}$ process we must add vertex function of ${}^3\text{He} \rightarrow \text{D} + \text{p}$ channel. Last vertex function is well known and is calculated in many approaches.

For calculation of differential cross sections of ${}^{6,7}\text{Li}(\text{d}, {}^{6,7}\text{Li}){}^2\text{H}$ and ${}^6\text{Li}({}^3\text{He}, \text{d}){}^7\text{Be}$ reactions it is necessary optical potential parameters for $\text{d}-{}^7\text{Be}$ and $\text{d}-{}^7\text{Li}$ interactions.

For calculation of $\text{d}-{}^7\text{Be}$ potential it is supposed that it is $\text{d}-{}^7\text{Li}$ potential with modified Coulomb's interaction. For test of such proposal it is calculated matrix elements of cluster folding potential. Potential of $\text{d}-{}^7\text{Be}$ interaction in framework of cluster model is integral:

$$V_{\text{d}-{}^7\text{Be}}(\vec{R}) = \left\langle \Psi_{\alpha\tau}(\vec{x}) \varphi_{pn}(\vec{z}) \middle| V \middle| \Psi_{\alpha\tau}(\vec{x}) \varphi_{pn}(\vec{z}) \right\rangle, \quad (1)$$

Where (x, z) – Jacoby set of coordinate for $\alpha\tau$ and pn system respectively; R - radius-vector, between mass center of incident deuteron and ${}^7\text{Be}$ nucleus, V - potential of interaction, which is sum of two particle potentials:

$$V = V_{p\alpha}(r_{13}) + V_{p\tau}(r_{14}) + V_{n\alpha}(r_{23}) + V_{n\tau}(r_{24}), \quad (2)$$

Where V_k – intercluster potential of interaction between i and j particles, which depend from relative distance r_{ij} .

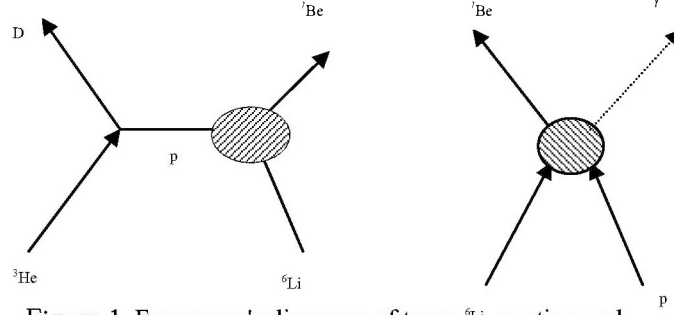


Figure 1. Feynmann's diagrams of trans ${}^6\text{Li}$ eaction and radiative capture reaction.

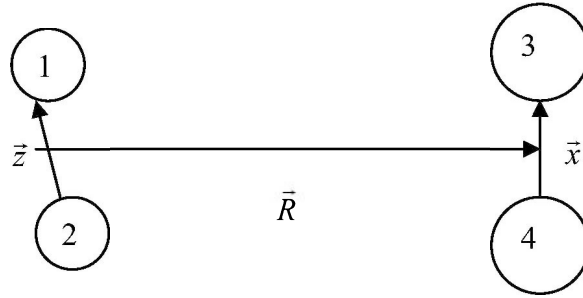


Figure 2. Coordinates in d - ${}^7\text{Be}$ system. 1 – p, 2 – n, 3 – α -particle, 3 – τ .

In cluster approach deuteron is considered as pn system and ${}^7\text{Be}$ nucleus as $\alpha\tau$ -system. For calculation of folding potential it is necessary convolute wave functions of pn and $\alpha\tau$ subsystems over intercluster potentials $p\alpha$, $p\tau$ and $n\tau$, $n\alpha$. In case of Gauss's potential using and Gauss's wave functions it is derived matrix elements which has analytical form. Matrix element is obtained within four body approach.

Wave functions of ${}^7\text{Be}$ nucleus are calculated within $\alpha\tau$ cluster model (Binding energy $E = -1.61$ MeV potential with core; $E = -1.57$ MeV deep potential), and also ${}^7\text{Li}$ nucleus as αt system. (binding energy $E = -2.44$ MeV potential with core and deep potential) with using of Buck's potential [6]. Wave function of pn system is calculated with Afnan-Tang potential [7] (binding energy $E = -2.16$ MeV, $E_{\text{exp}} = -2.23$ MeV) and with Argone 18 parameters potential for S-wave [8].

Pair potentials of $n-\alpha$ and $n-t$ subsystem were chosen in Gauss parameterization form from works [9;10], which are used for calculation of ${}^7\text{Li}$ -n elastic scattering (see Table 3).

Table. 3 Parameters of optical potentials for ${}^7\text{Li}$ -n elastic scattering differential cross section for энергии $E = 3.35$ MeV and $E = 4.83$ MeV. (S-channel spin of ${}^7\text{Li}$ -n system)

E , MeV	$S = 2$			$S = 1$		
	V_o	V_{sl}	χ^2	V_o	V_{sl}	χ^2
3.35	56.316	4.707	11.33	56.101	8.008	10.68
4.83	56.545	9.737	12.48	56.4	6.814	

Calculated folding potential was parameterized in Wood-Saxon form

$$V(R) = \frac{V_o}{1 + \exp\left(\frac{R - R_o}{a}\right)}$$

with set of parameters

$V_o = -73.8962$ MeV,

$R_o = 1.93253$ fm,

$a = 0.903627$ fm.

Depth of folding potential less than that of optical set, obtained from elastic scattering differential cross section analysis (see Table 1). And gives binding energy $E = -5.5$ MeV ($E_{\text{exp}} = -16.7$ MeV). Quality of description of elastic scattering at energies $E = 12$ MeV is less than that of optical potential.

Let us compare quality of Argone 18 parameters potential for S-wave. Our previous calculation for ${}^7\text{Li}$ -n folding potential and elastic scattering have showed that our choosing of αt , αn and $t n$ potential gives best description for ${}^7\text{Li}$ -n elastic scattering for energies of $E = 3.35$ MeV and $E = 4.83$ MeV, and this results obtained first in our work. In present time Argone 18 parameters potential for NN-system is one of the best potential. In 12 MeV for ${}^7\text{Li}$ -d there is only wide, not strong resonances. It is well known that pn-potentials has exchange forces and also D-state. From our calculation may be concluded that influence of none central parts of pn-potential play very important role in low energies.

For theoretical description of ${}^6\text{Li}({}^3\text{He}, d){}^7\text{Be}$ reaction cross section it is carried out optical potential analysis in ${}^6\text{Li}$ - ${}^3\text{He}$ channel in framework of standard optical model an coupled channel approach with Wood-Saxon parameterization, with volume and surface absorption. Parameters are reduced from adjusting of cross section to experimental data (Table.2.) [5].

Table 1. Optical potential parameters of interaction of deuteron with ${}^7\text{Be}$ nucleus

Канал	Набор	V , МэВ	r_V , ФМ	a_V , ФМ	W , МэВ	r_W , ФМ	a_W , ФМ	r_C , ФМ	Литер
${}^7\text{Be}+d$	A	138.7	1.02	0.86	65.3	1.31	0.76	1.25	[6]
	B				14.8	1.64	0.29	1.25	[6]

Set B - with surface absorption.

Table 2. Optical potential parameters of interaction of ${}^3\text{He}$ ion with ${}^6\text{Li}$ nucleus

A	$E_{{}^3\text{He}}$, МэВ		$-V$, МэВ	r_V , ФМ	a_V , ФМ	$-W$, МэВ	$r_{W(D)}$, ФМ	$a_{W(D)}$, ФМ	J_V , МэВ ФМ ³	χ^2/N
${}^6\text{Li}$	50	A	105.4	1.15	0.755	26.45	1.70	0.93	512	12.9
		A^*	113.8	1.15	0.748	23.33	1.23	0.80	547	23.0
	60	A	111.6	1.15	0.755	24.41	1.70	0.93	542	11.3
		A^*	110.2	1.15	0.750	22.50	1.22	0.80	531	28.0
	72	A	112.5	1.15	0.782	40.40	1.46	0.83	569	13.5
		A^*	107.2	1.15	0.752	21.60	1.21	0.80	518	25.0

* Potential with surface absorption.

3. In present work experimental data of elastic scattering of ${}^6\text{Li}({}^3\text{He}, {}^3\text{He}){}^6\text{Li}$ and ${}^7\text{Li}({}^2\text{H}, {}^2\text{H}){}^7\text{Li}$ nucleus is analyzed in framework of optical model. For ${}^7\text{Li}$ -d and ${}^7\text{Be}$ -d channel it is calculated cluster folding potential, where it is used pn and $\alpha t(\alpha\tau)$ wave functions. Using of gauss parameterization gives analytical matrix elements in four body approach. Obtained cluster folding potential was used for bind and scattering states description. Quality of description is worse than that of optical potential. From results it is concluded that none central parts of NN-potential play important role for low energy processes. Obtained potential parameters for ${}^7\text{Li}$ -d and ${}^7\text{Be}$ -d system will be used for calculation of ${}^6\text{Li}({}^3\text{He}, d){}^7\text{Be}$ reaction cross sections.

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