Scalar Meson Effects in Radiative $\phi \to \pi^0\eta\gamma$ Decay

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Abstract

We calculate the decay rate of the radiative $\phi \to \pi^0\eta\gamma$ decay using a phenomenological approach in which the contributions of ρ-meson, chiral loop and a_0-meson intermediate states are considered. We also analyze the invariant $\pi^0\eta$ mass spectrum for $\phi \to \pi^0\eta\gamma$ decay and compare it with experimental result.

The light scalar mesons with the vacuum quantum numbers have fundamental importance in understanding low energy QCD phenomenology and the symmetry breaking mechanisms in QCD. Although the nature and the quark substructure of the best known scalar mesons, isoscalars σ, $f_0(980)$ and isovector $a_0(980)$ have not be established yet [1, 2, 3], they are relevant hadronic degrees of freedom, and therefore their role in hadronic processes should be investigated.

The radiative decays of neutral vector mesons ρ^0, ω and ϕ into a single photon and a pair of neutral pseudoscalar mesons have been a subject of continuous interest theoretically and are becoming an area of active experimental research with the advent of high-luminosity, low energy e^+e^--machines such as Novosibirsk and Frascati that will make possible studying these rare decays with very small branching ratios. Such measurements may serve as tests for the theoretical ideas about the interesting mechanisms of these decays, as well as sheding light on the structure of the intermediate states involved in these decays.

In this paper, we follow a phenomenological approach and study the the mechanism of the radiative $\phi \to \pi^0\eta\gamma$ decay by considering ρ-pole vector meson dominance and chiral loop amplitudes as well as scalar a_0-pole amplitude.

The radiative decay process $\phi \to \pi^0\eta\gamma$ among other radiative decay processes of the type $V^0 \to P^0P^0\gamma$ where P and V belong to the lowest multiplets of vector (V) and pseudoscalar (P) mesons were studied by Fajfer and Oakes [4] using a low energy effective Lagrangian approach with gauged Wess-Zumino terms. They considered the contributions in which the virtual vector meson states dominate, and obtained the branching ratio for the decay as BR($\phi \to \pi^0\eta\gamma$)=5.18×10^{-5}. The contribution of intermediate vector mesons to $V^0 \to P^0P^0\gamma$ decays was later considered by Bramon et al. [5] using standard Lagrangians obeying the SU(3)-symmetry, and they obtained the result BR($\phi \to \pi^0\eta\gamma$)=5.4×10^{-6} for this decay. This result was not in agreement with the numerical result of Fajfer and Oakes [4], even if the initial expressions for the Lagrangians were the same. Bramon et al. [6] later considered $V^0 \to P^0P^0\gamma$ decays within the framework of chiral effective Lagrangians, and using chiral perturbation theory they calculated the branching

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ratios for various decays at the one-loop level. They showed that the one-loop contributions are finite and to this order no counterterms are required. In this approach the decay $\phi \to \pi^0\eta\gamma$ proceeds through charged-kaon loop, and by considering the amplitudes of both intermediate vector meson state and kaon-loop contributions they obtained the value BR($\phi \to \pi^0\eta\gamma$)=36×10^{-6} for the branching ratio of this decay. Radiative decays of ϕ meson were also investigated by Marco et al. [7] employing the techniques of chiral unitary theory developed earlier by Oller [8]. They included the final state interactions of two pions by summing the kaon-loops through Bethe-Salpeter equation and they obtained the branching ratio for the decay $\phi \to \pi^0\eta\gamma$ as $BR(\phi \to \pi^0\eta\gamma) = 0.87 \times 10^{-4}$. In a recent work Bramon et al. [9], by exploiting the complementary between Chiral Perturbation Theory and linear sigma model for this channel incorporated $a_0(980)$ resonance to the amplitude for $\phi \to \pi^0\eta\gamma$ decay and they predicted for the branching ratio $BR(\phi \to \pi^0\eta\gamma)$ a value in the range $(0.75 - 0.95) \times 10^{-4}$.

In our work, we include the scalar $a_0(980)$ meson resonance pole in the decay mechanism of $\phi \to \pi^0\eta\gamma$ in a phenomenological framework, and we assume that the radiative decay $\phi \to \pi^0\eta\gamma$ proceeds through the reactions $\phi \to \rho^0\pi^0 \to \pi^0\eta\gamma$, $\phi \to K^+K^-\gamma \to \pi^0\eta\gamma$, and $\phi \to a_0\gamma \to \pi^0\eta\gamma$ which we show diagramatically in Fig. 1, where the last diagrams in Fig. 1(b) results from the minimal coupling for gauge invariance.

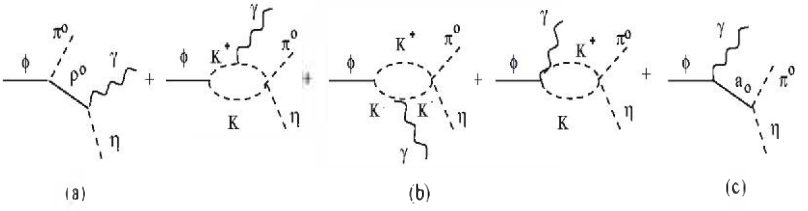


Figure 1: Diagrams for the decay $\phi \to \pi^0\eta\gamma$.

The $\phi\rho\pi$ vertex in Fig. 1(a) is conventionally described by the effective Lagrangian

$$\mathcal{L}^{eff}_{\phi\rho\pi} = g_{\phi\rho\pi}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\phi_\nu\partial_\alpha\rho_\beta\pi \quad , \tag{1}$$

and the coupling constant $g_{\phi\rho\pi}$ was determined by Achasov and Gubin using the data on the decay $\phi \to \rho\pi \to \pi^+\pi^-\pi^0$ [10] as $g_{\phi\rho\pi} = (0.811 \pm 0.081)\ GeV^{-1}$ [11]. The $\rho\pi\gamma$ vertex in Fig. 1(a) is described by the effective Lagrangian

$$\mathcal{L}^{eff}_{\rho\pi\gamma} = g_{\rho\pi\gamma}\epsilon^{\mu\nu\alpha\beta}\partial_\mu\rho_\nu\partial_\alpha A_\beta\eta \quad , \tag{2}$$

and the coupling constant $g_{\rho\eta\gamma}$ is then obtained from the experimental partial width of the radiative decay $\rho \to \eta\gamma$ [10] as $g_{\rho\eta\gamma} = (0.45 \pm 0.07)\ GeV^{-1}$.

We describe the ϕKK vertex in Fig. 1(b) by the effective Lagrangian

$$\mathcal{L}^{eff}_{\phi K^+K^-} = -ig_{\phi KK}\phi^\mu(K^+\partial_\mu K^- - K^-\partial_\mu K^+) \tag{3}$$

which results from the standard chiral Lagrangians in the lowest order of chiral perturbation theory [12]. The decay rate for the $\phi \to K^+K^-$ decay resulting from this Lagrangian is

$$\Gamma(\phi \to K^+K^-) = \frac{g^2_{\phi KK}}{48\pi}M_\phi\left[1 - \left(\frac{2M_K}{M_\phi}\right)^2\right]^{3/2} \quad . \tag{4}$$

We utilize the experimental value for the branching ratio $BR(\phi \to K^+K^-) = (0.492 \pm 0.007)$ for the decay $\phi \to K^+K^-$ [10], and determine the coupling constant $g_{\phi KK}$ as $g_{\phi KK} = (4.59 \pm 0.05)$. For the four pseudoscalar $KK\pi\eta$ amplitude, we use the result obtained in standard chiral perturbation theory by Bramon et al. [6] as

$$\mathcal{M}(K^+K^- \to \pi^0\eta) = \frac{\sqrt{3}}{4f_\pi^2}\left(M^2_{\pi^0\eta} - \frac{4}{3}M_K^2\right) \quad . \tag{5}$$

where $M_{\pi^0\eta}$ is the invariant mass of the $\pi^0\eta$ system, and in this order of chiral perturbation theory $f_\pi = 92.4$ MeV. We therefore obtain the amplitude for the diagram in Fig. 1(b) as

$$\mathcal{M} = -\frac{e\ g_{\phi KK}}{2\pi^2 i M_K^2}\left[(p\cdot k)(c\cdot u) - (p\cdot c)(k\cdot u)\right] I(a,b)\mathcal{M}(K^+K^- \to \pi^0\eta) \tag{6}$$

where (u,p) and (ϵ,k) are the polarizations and four-momenta of the ϕ meson and the photon, respectively, and $a = M_\phi^2/M_K^2$, $b = M^2_{\pi^0\eta}/M_K^2$ with the invariant mass of the final $\pi^0\eta$ system given by $M^2_{\pi^0\eta} = (q_1+q_2)^2 = (p-k)^2$. The loop function is defined as [13, 14]

$$I(a,b) = \frac{1}{2(a-b)} - \frac{2}{(a-b)^2}\left[f\left(\frac{1}{b}\right) - f\left(\frac{1}{a}\right)\right] + \frac{a}{(a-b)^2}\left[g\left(\frac{1}{b}\right) - g\left(\frac{1}{a}\right)\right] \tag{7}$$

where

$$f(x) = \begin{cases} -\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2 , & x > \frac{1}{4} \\ \frac{1}{4}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right]^2 , & x < \frac{1}{4} \end{cases}$$

$$g(x) = \begin{cases} (4x-1)^{\frac{1}{2}}\arcsin\left(\frac{1}{2\sqrt{x}}\right) , & x > \frac{1}{4} \\ \frac{1}{2}(1-4x)^{\frac{1}{2}}\left[\ln\left(\frac{\eta_+}{\eta_-}\right) - i\pi\right] , & x < \frac{1}{4} \end{cases}$$

$$\eta_\pm = \frac{1}{2x}\left[1 \pm (1-4x)^{\frac{1}{2}}\right] . \tag{8}$$

We describe the $\phi a_0\gamma$ and $a_0\pi^0\eta$ vertices in Fig. 1(c) by the effective Lagrangians

$$\mathcal{L}^{eff}_{\phi a_0\gamma} = \frac{e}{M_\phi} g_{\phi a_0\gamma}[\partial^\alpha\phi^\beta\partial_\alpha A_\beta - \partial^\alpha\phi^\beta\partial_\beta A_\alpha]a_0 \tag{9}$$

and

$$\mathcal{L}^{eff}_{a_0\pi\eta} = g_{a_0\pi\eta}\vec{\pi}\cdot\vec{\pi}a_0 \quad , \tag{10}$$

which also serve to define the coupling constants $g_{\phi a_0\gamma}$ and $g_{a_0\pi\eta}$. The decay rates for the $\phi \to a_0\gamma$ and the $a_0 \to \pi^0\eta$ decays resulting from the above Lagrangians are

$$\Gamma(\phi \to a_0\gamma) = \frac{\alpha}{24\pi}\frac{(M_\phi^2 - M_{a_0}^2)^3}{M_\phi^5}g^2_{\phi a_0\gamma} \quad , \tag{11}$$

and

$$\Gamma(a_0 \to \pi^0\eta) = \frac{g^2_{a_0\pi\eta}}{16\pi M_{a_0}}\sqrt{\left[1 - \frac{(M_{\pi^0}+M_\eta)^2}{M_{a_0}^2}\right]\left[1 - \frac{(M_{\pi^0}-M_\eta)^2}{M_{a_0}^2}\right]} \quad , \tag{12}$$

respectively. We use the value $\Gamma_{a_0} = (0.069 \pm 0.011)$ GeV [15], and obtain the coupling constant $g_{a_0\pi\eta}$ as $g_{a_0\pi\eta} = (2.32 \pm 0.18)$ GeV. On the other hand, there are no direct experimental results relating to $\phi a_0\gamma$-vertex, but only an upper limit of the branching ratio of the decay $\phi \to a_0\gamma$ is quoted in Review of Particle Properties as $BR(\phi \to a_0\gamma) < 5 \times 10^{-3}$ [10]. However, the coupling constant $g_{\phi a_0\gamma}$ has been calculated [16] employing QCD sum rues and utilizing $\omega\phi$-mixing by studying the three point $\phi a_0\gamma$-correlation function as $g_{\phi a_0\gamma} = (0.11 \pm 0.03)$. Furthermore, the photoproduction cross section of ρ^0-meson on proton targets near threshold has been shown [17] to be mainly given σ-exchange and assuming vector meson dominance of the electromagnetic current the value of the coupling constant $g_{\rho\sigma\gamma} = 2.71$ was deduced. In the study of the structure of ϕ meson photoproduction amplitude on nucleons near threshold based on one-meson exchange and Pomeron-exchange mechanism [18], this value of the coupling constant $g_{\rho\sigma\gamma}$ was used to calculate the coupling constant $g_{\phi a_0\gamma}$ by invoking unitary symmetry arguments as $g_{\phi a_0\gamma} = 0.16$ by assuming that σ, f_0, and a_0 are members of a unitary nonet. Therefore, we use these available values in the literature for the coupling constant $g_{\phi a_0\gamma}$ in our calculation of the decay rate of the $\phi \to \pi^0\eta\gamma$ decay.

In our calculation of the invariant amplitudes we make the replacement $p^2 - M^2 \to p^2 - M^2 + iM\Gamma$ in a_0 and ρ^0 propagators and use the experimental value $\Gamma_\rho = (150.2 \pm 0.8)$ MeV [10] for ρ^0 meson, and we write the energy dependent width of the a_0 resonance as

$$\Gamma^{a_0}(q^2) \;=\; \Gamma^{a_0}_{\pi^0\eta}(q^2)\;\theta(\sqrt{q^2} - (M_{\pi^0} + M_\eta)) \quad , \tag{13}$$

where

$$\Gamma^{a_0}_{\pi^0\eta}(q^2) = \frac{g^2_{a_0\pi\eta}}{16\pi(q^2)^{3/2}}\;\sqrt{[q^2 - (M_{\pi^0} + M_\eta)^2][q^2 - (M_{\pi^0} - M_\eta)^2]} \quad . \tag{14}$$

We express the invariant amplitude $\mathcal{M}(\mathrm{E}_\gamma, E_1)$ for the decay $\phi \to \pi^0\eta\gamma$ as $\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b + \mathcal{M}_c$ where $\mathcal{M}_a$, $\mathcal{M}_b$, and $\mathcal{M}_c$ are the invariant amplitudes resulting from the diagrams (a), (b), and (c) in Fig. 1. This way we take the interference between different reactions contributing to the decay $\phi \to \pi^0\eta\gamma$ into account. Then the differential decay probability for $\phi \to \pi^0\eta\gamma$ for an unpolarized ϕ meson at rest is given as

$$\frac{d\Gamma}{dE_\gamma dE_1} = \frac{1}{(2\pi)^3}\;\frac{1}{8M_\phi}\;\mid \mathcal{M} \mid^2, \tag{15}$$

where E_γ and E_1 are the photon and pion energies respectively. We perform an average over the spin states of ϕ meson and a sum over the polarization states of the photon. The decay width $\Gamma(\phi \to \pi^0\eta\gamma)$ is then obtained by integration

$$\Gamma = \int_{E_{\gamma,min.}}^{E_{\gamma,max.}} dE_\gamma \int_{E_{1,min.}}^{E_{1,max.}} dE_1 \frac{d\Gamma}{dE_\gamma dE_1} \tag{16}$$

where the minimum photon energy is $E_{\gamma,min.} = 0$ and the maximum photon energy is given as $E_{\gamma,max.} = [M^2_\phi - (M_{\pi^0} + M_\eta)^2]/2M_\phi$. The maximum and minimum values for pion energy E_1 are given by

$$\frac{1}{2(2E_\gamma M_\phi - M^2_\phi)}[-2E^2_\gamma M_\phi - M_\phi(M^2_\phi + M^2_{\pi^0} - M^2_\eta) + E_\gamma(3M^2_\phi + M^2_{\pi^0} - M^2_\eta)$$
$$\pm E_\gamma\sqrt{4E^2_\gamma M^2_\phi + M^4_\phi + (M^2_{\pi^0} - M^2_\eta)^2 - 2M^2_\phi(M^2_{\pi^0} + M^2_\eta) + 4E_\gamma M_\phi(-M^2_\phi + M^2_{\pi^0} + M^2_\eta)}\,].$$

If we use the value $g_{\phi a_0\gamma} = 0.11$ for the coupling constant $g_{\phi a_0\gamma}$ that was determined from light cone QCD sum rules [16] we obtain the branching ratio for the decay $\phi \to \pi^0\eta\gamma$ in our phenomenological framework as $\mathrm{BR}(\phi \to \pi^0\eta\gamma) = 0.67 \times 10^{-4}$. The value of the coupling constant $g_{\phi a_0\gamma} = 0.16$ as determined by assigning σ, f_0 and a_0 mesons into a unitary nonet and using the value $g_{\rho\sigma\gamma} = 2.71$ [18] results in the branching ratio $\mathrm{BR}(\phi \to \pi^0\eta\gamma) = 0.79 \times 10^{-4}$ for the $\phi \to \pi^0\eta\gamma$ decay in our approach.

The very accurate data that have been obtained from Novosibirsk SND [19] and CMD-2 [20] collaborations give the following branching ratios for $\phi \to \pi^0\eta\gamma$ decay: $\mathrm{BR}(\phi \to \pi^0\eta\gamma) = (0.88 \pm 0.14 \pm 0.09) \times 10^{-4}$ [19] and $\mathrm{BR}(\phi \to \pi^0\eta\gamma) = (0.90 \pm 0.24 \pm 0.10) \times 10^{-4}$ [20], where the first error is statistical and the second one is systematic. If we compare our results for the branching ratio $\mathrm{BR}(\phi \to \pi^0\eta\gamma)$ with the experimental values for this brancing ratio, we may conclude that our phenomenological description of the reaction mechanism of the $\phi \to \pi^0\eta\gamma$ decay is physically feasible. In order to test our approach for the reaction mechanism of the $\phi \to \pi^0\eta\gamma$ decay further, we plot the invariant mass distribution $dBR/dM_{\pi^0\eta} = (M_{\pi^0\eta}/M_\phi)dBR/dE_\gamma$ for the radiative decay $\phi \to \pi^0\eta\gamma$ in our phenomenological approach using the values of the coupling constant $g_{\phi a_0\gamma} = 0.11$ in Fig 2 and $g_{\phi a_0\gamma} = 0.16$ in Fig. 3 and compare them with the experimental invariant $\pi^0\eta$ mass spectrum obtained by SND Collaboration [19]. We see that although our approach gives a reasonable prediction for he spectrum over most of the range of the invariant mass $M_{\pi^0\eta}$, the expected enhancement in its higher part due to the contribution of a_0 resonance is not produced. We may therefore conclude that the problem of incorporating the scalar mesons into the reaction mechanisms of radiative ϕ meson decays cannot be solved in a straightforward fashion and requires further intensive studies.

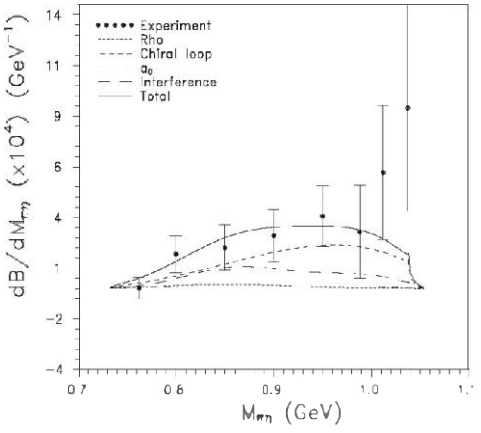


Figure 2: The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{\phi a_0\gamma} = 0.11$. The contributions of different terms are indicated.

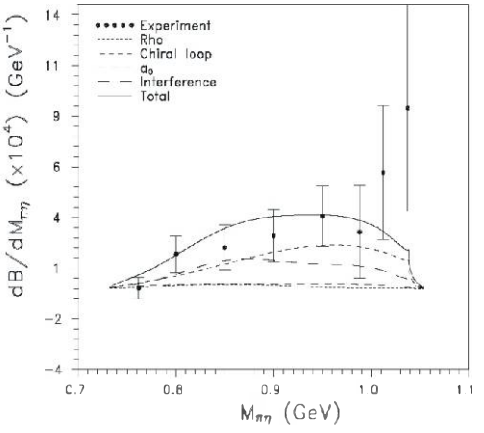


Figure 3: The $\pi^0\eta$ invariant mass spectrum for the decay $\phi \to \pi^0\eta\gamma$ for $g_{\phi a_0\gamma} = 0.16$. The contributions of different terms are indicated.

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