THE RESISTIVE MAGNETOHYDRODYNAMICS (MHD) IN THERMONUCLEAR DISCHARGES :THEORY AND EXPERIMENT

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ABSTRACT.-
It has been understood that thermonuclear plasma confinement and behaviour of the plasma was mostly depend upon the magnetic surfaces on which the confining magnetic field and the plasma current lie. However magnetic surfaces are not stable and prone to the magnetohydrodynamic(MHD) stabilities and reconnection and finaly they lead to the magnetic island formation. Resistive magnetohydrodynamics and instabilities are reviewed both experimentally and theoretically, among them; tearing mode and mode locking phenomena, topological relaxation (sawteeth oscillation), magnetic island overlap and plasma minor and major distruption.are investigated. The tearing mode growth rate are determined. Magnetic field reconncection on the rational surfaces and magnetic island formation, magnetic island growth rate and plasma confinement relation and energy confinemenet degradation are emphasized: The island width and island evaluation equations are derived. Performance limitation and MHD activities and mode structure m=0,n=0,m=1,n=1 and their sidebands and overlap are reviwed. Plasma distruptive phenomena with up to date experiments are presented.

1.INTRODUCTION

Plasmas are classified in terms of resistivity η, if $\sigma\to\infty,\eta\to 0$ and MHD is called ideal MHD, if resistivity is finite, called <u>RESISTIVE MHD</u>. Thermonuclear plasmas $\mathbf{T \geq 10keV, n \approx 10^{19} cm^{-3}, B = 2-3T}$ are confined in a Toroidal Vacuum Chamber (TOKAMAK) by balancing the plasma kinetic pressure with the magnetik pressure and the efficiency of the plasma confinement is measured by the ratio of them and called the plasma beta(β). The poloidal field is much smaller than toroidal field.From Ohm's law and resistivity current and temperature profile can be deduced:poloidal cross section and follow the field line while ıt penetrates into this surface. If field line comes to on itself after m-times around the torus and n-times the poloidal transit we denote that q-value or safety factor and is given as q=m/n on mode rational surfaces.

2.THE RESISTIVE MAGNETOHYDRODYNAMIC EQUATIONS

As we pointed out the ideal model of the plasma is to solve the ideal MHD and Maxwell electromagnetic equation. imple case is the electrostatic modes and incompressible mode:

$$\vec{\mathbf{E}} = -\vec{\nabla}\phi,\ \ \vec{\nabla}\bullet\vec{\mathbf{V}} = 0 \Rightarrow \frac{\partial\rho_m}{\partial t} = -\vec{\mathbf{V}}\bullet\vec{\nabla}\rho_m - \vec{\nabla}\phi + \vec{\mathbf{V}}\times\vec{\mathbf{B}} = 0 \xrightarrow{\vec{\nabla}\times} \vec{\mathbf{V}}_\perp = \frac{\vec{\mathbf{B}}\times\vec{\nabla}\phi}{\mathbf{B}^2} \quad (1)$$

The perturbation of equilibrium in a magnetized plasma with perturbed quantities:

$$\rho_m \rightarrow \rho_{m0} + \tilde{\rho}, \vec{V} \rightarrow \tilde{\vec{V}}, \vec{B} \rightarrow \vec{B}_0, \vec{J} \rightarrow \vec{J}_0 + \tilde{\vec{J}} p \rightarrow p_0 + \tilde{p}, \phi \rightarrow \tilde{\phi} \qquad (2)$$

With Linearization procedure , we obtain the linearized ideal electrostatic MHD equations:

$$\textbf{density con.:}\ \frac{\partial\tilde{\rho}_m}{\partial t} = -\tilde{\vec{V}} \bullet \vec{\nabla}\rho_{m0}, \textbf{Mom. den.}:\rho_{m0}\frac{\partial\tilde{\vec{V}}}{\partial t} = \tilde{\vec{J}} \times \vec{B}_0 - \vec{\nabla}\tilde{p} - \tilde{\rho}_m\vec{\nabla}G$$
$$\textbf{Energy con.:}\ \tilde{p} = (\Gamma p_0 / \rho_{m0})\tilde{\rho}_m, \ \vec{V}_\perp = \frac{\vec{B}_0 \times \vec{\nabla}\tilde{\phi}}{B_0^2} \qquad (3)$$

The strategy here extract the perturbed current component and use them in the quasi neutrality condition. The Resistive Ohm's Law is the cause of magnetic reconnection: Combining the Faraday induction Law and Resistive Ohm's Law[1]. Magnetic convection and magnetic difusion are competing with their time scale:

$$\textbf{Alfvenic (convective) t}\tau_A = \frac{a}{V_A}[\mu\,\text{sec}], \ \textbf{diffusion t} \ \ \tau_R = a^2/(\eta/\mu_0)[\text{m sec}] \qquad (4)$$

The ratio of two is called the magnetic Reynlds or Lundquist number:

$$S \approx \frac{\vec{\nabla}\times(\tilde{\vec{V}}\times\vec{B}_0}{\frac{\eta}{\mu_0}\nabla^2\tilde{\vec{B}}} = \frac{\tau_R}{\tau_A} = \frac{a^2/(\eta/\mu_0)}{a/V_A} \rightarrow \frac{\textbf{Convection}}{\textbf{Diffusion}} \qquad (5)$$

Since this number is large for conventional tokamaks the , resistivity is effective on the rational surfaces(i.e q=m/n): the model is conceptualized for shear slab. We obtain the paralel wave number:

$$k_{II} = \frac{n}{R}\frac{B_\phi}{B} - \frac{m}{r}\frac{B_\theta}{B}, \textbf{safety factor } q = \frac{r}{R}\frac{B_\phi}{B_\theta} \qquad (6)$$

With magnetic shear and shear length:

$$s = r\frac{1}{q}\frac{dq}{dr}, L_s = R\frac{q(r_s)}{s(r_s)}, x = r - r_s, k_{II} = \left.\frac{mx}{rL_s}\right|_{r=r_s} = \frac{k_\theta x}{L_s}, \ k_\theta = \left.\frac{m}{r}\right|_{r_s} \qquad (7)$$

Sheared slab model is represented with :

$$\vec{B} = \vec{B}_0(\hat{z} + \hat{y}\frac{x}{L_s}) \rightarrow k_{II}(x) = \vec{k}\bullet\vec{B} = (k_y\hat{y} + k_z\hat{z})\bullet\vec{B}, B_0(k_z + \frac{x}{L_s}k_y) \qquad (8)$$

The resistive layer thickness can be estimated by scaling the gradient terms as follows:

$$\frac{V_A^2}{\gamma}\frac{k_y^2\delta^2}{L_s^2} \approx \frac{\eta}{\mu_0}\frac{1}{\delta^2} \Rightarrow \frac{\delta}{a} \approx \left[\frac{(\gamma\tau_A)}{S}\left(\frac{L_s}{a}\right)^2\left(\frac{1}{k_y^2a^2}\right)\right] \qquad (9)$$

The resistive plasma equations with Ohm's Law and perpendicular current:

$$\vec{E} \times \vec{V} \times \vec{B} = \eta_{II}\vec{J}_{II}, \ \eta_\perp \rightarrow 0 \qquad (10)$$

$$\vec{J}_\perp = \frac{1}{B^2}\vec{B}\times(\vec{\nabla}p + \rho_m\frac{d\vec{V}}{dt} + \rho_m\vec{\nabla}G)\,\text{diamagnetic+polarization} \tag{11}$$

The electric and magnetic field are written in terms of the vector potential:

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial A}{\partial t},\ \vec{B} = \vec{\nabla}\times\vec{A} \to \nabla^2 A = -\mu_0 j \tag{12}$$

Chose oscillation only perpendicular direction:

$$\vec{A}_z = \vec{A}_{z0} + \tilde{A}_z \Rightarrow \vec{B}_\perp = \hat{z}\times\vec{\nabla}\tilde{A}_z \tag{13}$$

With $\omega<<\omega p$, $k\lambda D<<1$,quasineutrality $\rho\to 0$,slow waves $\omega/ck<<0$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial\vec{A}}{\partial t}, \vec{B} = \vec{\nabla}\times\vec{A}, -\nabla^2\vec{A} = \mu_0\vec{J},\ \ \vec{\nabla}\bullet\vec{J} = 0 \tag{14}$$

Ohm's Law can be converted into: $\frac{dA_z}{dt} = -\frac{\partial\phi}{\partial z} - \eta J_{II} + E_{II}^{A}$ (15)

3.RESISTIVE MHD INSTABILITIES

Resistive MHD instabilities are based upon magnetic reconnection of magnetic surfaces. They are grouped into different modes for their driving forces,such as resistive-g mode,rippling mode and tearing mode. Beyond that mode coupling , mode locking and sawtooth oscillation are major causes of plasma confinement degradation and/or even distruption.

Resistive g-mode:The driving force of this mode is the radial pressure gradient. Perturbation can be modelled and perturbed velocity and pressure extracted:

Upon substitution perturbed pressure and current equation into quasineutrality equation one can obtain($G\to LnB0$)

$$\left[\frac{\rho_m}{B_0^2}\left(\frac{d}{dx^2} - k_y^2\right)\frac{1}{i\omega\eta} + \frac{k_y^2x^2}{L_s^2} - \frac{2k_y^2}{\omega^2B_0^2}\frac{dp_0}{dx}\frac{dG}{dx}\right]\hat{\phi}(x) = 0 \tag{16}$$

The solution of this equation with the asumption: $\frac{\partial^2\phi}{\partial x^2}\rangle\rangle k_y^2\phi$ and by de-dimensionalize with

$$: X \equiv \frac{x}{\delta_R}, \delta_R = \left[\frac{\gamma\tau_A L_s^2}{Sk_y^2}\right]^{1/4},\ \frac{d^2\hat{\phi}}{dX^2} + \left[\frac{2k_y^2\delta_R^2}{-\omega^2\rho_{m0}}\frac{dp_0}{dx}\frac{dG}{dx} - X^2\right]\hat{\phi} = 0 \tag{17}$$

This leads to ,with the lowest order eigen value n=0, the growth rate and witdh

$$1 = \left[\frac{2k_y^2\delta_R^2}{\left(-\omega^2\rho_m\right)}\frac{dp_0}{dx}\frac{dG}{dx}\right], \gamma\tau_A = \frac{\left(k_yL_s\right)^{2/3}}{S^{1/3}(\gamma_H\tau_A)^{-4/3}}, \frac{\delta_R}{L_s} = \frac{(\gamma_H\tau_A)^{1/3}}{S^{1/3}(k_yL_s)^{1/3}} \tag{18}$$

Rippling Mode: This is temperature i.e. resistivity perturbation. Then Ohm's law is written in terms of resistivity perturbation and current density is extracted:

$$-(\vec{b}_0\bullet\vec{\nabla})\tilde{\phi} - \eta_0\tilde{J}_{II} - \tilde{\eta}J_{II0} = 0, \tilde{J}_{II} = -\frac{(\vec{b}_0\bullet\vec{\nabla})\tilde{\phi}}{\eta_0} - \frac{\tilde{\eta}}{\eta_0}J_{II} \tag{19}$$

The quasineutrality equation without pressure gradient and under the same assumption
Then our differential equation :

$$\frac{\rho_{m0}}{B_0^2}\frac{d^2\tilde{\phi}}{dx^2}+\left[\frac{1}{i\omega\eta_0}\frac{k_y^2}{L_s^2}x^2+\frac{k_y^2}{\omega}\frac{J_{II0}}{\eta_0}\frac{1}{B_0}\frac{d\eta_0}{dx}x\right]\tilde{\phi}=0 \tag{20}$$

Thedimensionless form : $\frac{d^2\tilde{\phi}}{dX^2}+\left[-X^2+X\Delta\right]\tilde{\phi}=0,\Delta=\left(\frac{k_y\delta_R}{\omega\tau_A}\right)^2\left(\frac{a^2}{L_s^2}\right)\frac{\delta_R}{\eta_0}\frac{d\eta_0}{dx}$ (21)

Shifting the origion from X=0 toX=Δ/2 and redefineX-Δ/2 AS X

$$\frac{d^2\tilde{\phi}}{dX^2}+\left[-X^2+X\Delta-\frac{\Delta^2}{4}+\frac{\Delta^2}{4}\right]\tilde{\phi}=0,\frac{d^2\tilde{\phi}}{d\chi^2}+\left[\frac{\Delta^2}{4}-\chi^2\right]\tilde{\phi}=0,\ 2n+1=\frac{\Delta^2}{4} \tag{22}$$

The growing rate : $\gamma\tau_A=\frac{(k_a a)^{2/5}}{S^{3/5}(L_s/a)^{2/5}}\left[\frac{a}{\eta_0}\frac{d\eta_0}{dx}\right]^{4/5}\frac{1}{[4(2n+1)]^{4/5}}$ (23)

<u>*Tearing mode*</u>: The driving force for free energy of this mode is the current density gradient(dJ/dr). The perturbation has perpendicular magnetic field component the potantiel perturbation and vector potential.

$$\phi=\tilde{\phi},A_z=A_{z0}+\tilde{A}_{z0},\tilde{A}_{z0}=-\frac{x^2}{2L_s}B_0,p=p_0+\tilde{p} \tag{24}$$

The magnetic perturbation and current densitycan be written in terms of the vector potential:İ

$$\vec{J}=-\frac{ik_y x}{\eta_0 L_s}\tilde{\phi}+\frac{i\omega}{\eta_0}\tilde{A}_z,\tilde{\vec{B}}_\perp=-\hat{z}\times\vec{\nabla}\tilde{A}_z=-\hat{z}\times\left[\frac{\partial\tilde{A}_z}{\partial x}\hat{x}+\frac{\partial\tilde{A}_z}{\partial y}\right],ik_yA_z\hat{x}-\frac{\partial\tilde{A}_z}{\partial x}\hat{y} \tag{25}$$

Upon insertion of current density and magnetic field and seperate the two potentiels on each side the equation, one can obtain:

$$-i\omega\frac{\rho_{m0}}{B_0^2}\frac{d^2\phi}{\partial x^2}-\frac{k_y^2x^2}{\eta_0L_s^2}\tilde{\phi}=-\left(\frac{\omega k_y}{\eta_0L_s}x-\frac{ik_y}{B_0}\frac{dJ_{II}}{dx}\right)\tilde{A}_z \tag{26}$$

$$\Delta'\equiv\frac{\left[\frac{d\tilde{A}_z}{dx}\right]}{\tilde{A}_z}=\frac{\left.\frac{d\tilde{A}_z}{dx}\right|_{x\rangle 0}-\left.\frac{d\tilde{A}_z}{dx}\right|_{x\langle 0}}{\tilde{A}_z}\propto\frac{1}{length},(\Delta'a)=-\frac{i\omega\delta_R a}{\frac{\eta_0}{\mu_0}}I\xrightarrow{\delta_R\equiv\left[\gamma\tau_A L_s^2/Sk_y^2\right]^{1/4}}$$

$$\textbf{Unstability condition : }\Delta'\rangle 0,\textbf{growth rate: }\gamma\tau_A=\left(\frac{\Delta'a}{I}\right)^{4/5}\frac{(k_ya)^{2/5}}{S^{3/5}(L_s/a)^{2/5}} \tag{27}$$

$$\textbf{resistive layer width: }\frac{\delta_R}{a}=\left(\frac{\Delta'a}{I}\right)\left[\frac{L_sa}{Sk_ya}\right]^{2/5}$$

4.EXPERIMENT: MAGNETIC ISLAND FORMATION

Magnetic reconnection and tearing mode plays the causal role of the internal distruption, which is q=m/n=1/1, Mirnow oscillation which is q=m/n=2/1,3/1,..., and major distruption which is the coupling of 2/1 and 3/2 modes on the mode rational surfaces. The formation of magnetic islands on the magnetic surfaces progress from kinks of the surfaces, then tearing of the surfaces and low order of magnetic islands are formed; as a consequence energy confinement is reduced even loss of confinement, i.e distruption, such as m/n=2/1.

Mirnov oscillations: this is m/n=2/1 tearing mode during the steady state of the discharge. The magnetic island widh and growing rate can be found as follows:

$$\mathbf{W = 4\sqrt{\frac{\widetilde{B}_{rmn} r_{smn}}{B_\theta n q'}}, \frac{dW}{dt} = \frac{\eta}{\mu_0}\left[\Delta'(W) - \alpha W\right]} \tag{28}$$

Sawtooth oscillations: This can be atributed m/n=1/1 tearing mode and formation of magnetic island and its grow. In the central region magnetic fields reconnect, the hot core region displaces the cold edge region and electron temperature starts to oscillate. This is the case when q-value drops below 1 in the core. This can be observed by soft X-ray emission from the core.

Double tearing mode: This occurs during the initial phase of the discharge,i.e in the progress of current penetration. Tearing mode is observed at two different radial position,that is at two different rational surfaces.

*Major distruption:*As we saw above the resistive modes are the nonlinear growth of island on the rational surfaces m/n=1/1,2/1,3/2,... with the perturbation modelled by exp(imθ-inφ) depending on the current J(r) and safety factor profile q(r). Among those the major distruption,i.e loss of confinement/ or current of the discharge, is the overlap of the 2/1 and 3/2 magnetic islands. The criterion for that:

$$W = \sqrt{\frac{\widetilde{B}_r r_s}{B_\theta q'}} \rangle d_{island\,\mathrm{separation}} \Rightarrow magnetic\ \mathrm{field\,,stochastictiy} \Rightarrow \mathrm{negative\ voltage\ spikes} \Rightarrow \mathrm{J(r)}$$
$$\mathrm{current\ loss}$$

5.CONCLUSION

Utilization of Thermonuclear power is close to final stage, as long as resistive instabilites are under control. The stabilization and control of these instabilities require more research dominantly more experiment which comply fusion condition. This is the current obstacle which should be resolved.The sawtooth oscillation and control will come next.

REFERENCES

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