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NEW THEORY OF GRAVITATION

Basic Principles, Correspondence Structure, Experimental Confirmation*

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ABSTRACT

A necessary and sufficient condition for a space-time theory of gravitation to recover the many-body Newtonian orbits and their perturbations is shown to be a modification of the field equations of general relativity by an additive term t_μ^ν which represents the stress-energy tensor of the gravitational field.

ÖZET

Bu yazıda zaman-mekan teorisinin gezegenler ve onların pertürbasyonlarını kapsayan Newton dinamiğine icra olabilmesi için genel görelilik teorisi alan denklemine yerçekim stress-enerjisi t_μ^ν nün ilave edilmesi gerek ve yeterliği gösterilmiştir.

INTRODUCTION

Given an arbitrary space-time geometry $g_{\mu\nu}$ one can have in this geometry, particles with rest-mass m and energy-momenta $p_\mu = mu_\mu$ without questioning how the $g_{\mu\nu}$ is generated. One can also define the total energy momentum $P_\mu = \Sigma p_\mu$ (of the particles plus the field as the latter is imbedded in the definition of $u_\mu = dx_\mu/ds$) and the total Action $A = \Sigma \int p_\mu dx^\mu = \Sigma \int m ds$, $\delta A = 0$, hence prove that each particle moves on a geodesic /1/

$$\frac{du_\mu}{ds} = \tfrac{1}{2}\partial_\mu g_{\alpha\beta} u^\alpha u^\beta$$

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both in magnitude and in functional form. The additive constant $K = \Sigma mc^2$ is irrelevant to the Newtonian dynamics but in relativistic theory it recovers the special relativistic rest energy again both in magnitude and in form (the units are chosen so that $4\pi G = c = 1$). We can thus conclude that the theory under consideration recovers the full content of the Newtonian dynamics as a correspondence limit and therefore, predicts correctly the Newtonian orbits and their perturbations. Included in such predictions is the observationably confirmed 532" per century Newtonian component of the advance of the perihelion of Mercury.

As is well known, the perihelion of Mercury advances 575" per century of which 532" is a Newtonian perturbative effect (mainly due to Jupiter, Venus and Earth) and only the remaining 43" is of relativistic origin. The theory under consideration also predicts correctly this relativistic part because the PPN (parametric post-Newtonian) coefficients are $\alpha = \beta = \gamma = 1$. In fact, from these coefficients one can also infer that other relativistic effects such as the red-shift, light-bending and radar echo-delay are likewise correctly predicted.

Clearly, in order to predict correctly the full Newtonian range of phenomena plus the relativistic corrections such as mentioned, it is sufficient to consider the field equations with a source $\tau_\mu^\nu + t_\mu^\nu$ as described. The question arises as to whether this is also necessary. For example, the usual general relativity claims to do the same without introducing t_μ^ν. To investigate this problem let us now write the divergence identity without a t_μ^ν. One has $(\sqrt{-g})^{-1}\,\partial_\nu(\sqrt{-g}\tau_\mu^\nu) - \frac{1}{2}\,\partial_\mu g_{\alpha\beta}\tau^{\alpha\beta} = 0$. Because of the last term this condition is not the same as before but it will certainly conserve something and do something to p_μ. But we can find out what it will do by integrating the equation. Let the effect of the divergence condition above be $P_\mu \to Q_\mu$ where Q_μ is conserved. We have the form

$$\int \sqrt{-g} D_\nu \tau_\mu^\nu d\Omega = Q_\mu - Q_\mu^{(0)} = 0$$

where we are interested in Q_μ and its functional form. We can perform the integration by replacing $\frac{1}{2}\partial_\mu g_{\alpha\beta}\tau^{\alpha\beta}$ with $\sigma du_\mu/ds$ from the (geodesic) equation of motion.

It can be shown that (see the summary at the end of the paper) these equations constitute a new theory of gravitation where the source of the geometric curvatures is the sum of the matter stress-energy τ_μ^ν and the quantity t_μ^ν which turns out to be the gravitational field stress-energy. Many exact solutions to the field equations of this theory are already known. For example, a general "iteration" solution

$$g_{\mu\nu} = (\tilde{\eta}e^{2(\phi-2\tilde{\phi})})_{\mu\nu}$$

where $\tilde{\eta}$ is a Lorentz metric, $\tilde{\phi} = \phi_\mu^\nu$, $\partial_\nu\phi_\mu^\nu = 0$, ϕ = trace $(\tilde{\phi})$ and where ϕ_μ^ν satisfies the generalized Poisson equation

$$\Box^2\phi_\mu^\nu = \tau_\mu^\nu$$

provides exact solutions whenever $g_{\mu\nu}$ is expressible in terms of known functions. In this category is the low velocity limit $u^i \to 0$, $u_0u^0 \to 1$, $\phi_\mu^\nu \to \phi_0^0 = \phi$, $\partial_0\phi = 0$ which we shall use in our forthcoming analysis of the problem of planetary dynamics. In this case the general solution gives /1,3/

$$g_{00} = e^{-2\phi} \quad , \quad -g_{ii} = e^{2\phi}$$

$$\phi(x) = \sum_j \frac{m_j}{|x-x_j|}$$

$$t_\mu^\nu = -\partial_\mu\phi\partial^\nu\phi + \tfrac{1}{2}\partial_\mu^\nu\,\partial^\lambda\phi\partial_\lambda\phi$$

$$H = P_0 = \sum_\ell [e^{-\phi}\sqrt{(m^2+e^{-2\phi}p^2)}]_\ell$$

Note that these expressions have a marked similarity to the Newtonian theory. In fact, the total Hamiltonian not only gives exactly the geodesic equations of motion $du_\mu/ds = -2u_0u^0\partial_\mu\phi^0 + \partial_\mu\phi$ of the space $g_{00} = e^{-2\phi}$, $-g_{ii} = e^{2\phi}$ but also recovers in the limit $u_0u^0 \to 1$ the many-body Newtonian Hamiltonian

$$H_N = \sum_k [\frac{p_k^2}{2m_k} - \tfrac{1}{2}\sum_j \frac{m_km_j}{|x_k-x_j|}]_k + K$$

The latter can be done either by varying the action or applying the canonical equations of motion to the Hamiltonian $H = P_0$. Furthermore, one can put the total energy-momentum into its integral form $P_\mu = \int_V \sqrt{-g}\tau_\mu^\nu dV_\nu$ where $\tau_\mu^\nu = \sigma u_\mu u^\nu$, $\sqrt{-g}\sigma = \Sigma_j m_j \delta(x-x_j)$. However, despite all of these one cannot yet prove that the total energy-momentum P_μ is conserved. (For example, if $g_{\mu\nu}$ is partly or wholly external, P_μ is not necessarily conserved.) In order to conserve P_μ a further condition, $\partial_\nu(\sqrt{-g}\tau_\mu^\nu) = 0$, must be satisfied, since it is only then that Gauss' theorem yields /2/

$$\int \partial_\nu(\sqrt{-g}\tau_\mu^\nu)d\Omega = \int_V \sqrt{-g}\tau_\mu^\nu dV_\nu - \int_{V_0} \sqrt{-g}\tau_\mu^\nu dV_\nu = 0$$

implying $P_\mu = P_\mu^{(0)}$ is conserved.

Clearly the requirement that P_μ will become conserved without otherwise altering or contradicting its above definition (for example without altering its magnitude and its functional form), imposes certain conditions on $g_{\mu\nu}$ since it can no longer be as arbitrary as it was taken in the beginning. Let us then consider the equations of $g_{\mu\nu}$ first in a rather general form /1/

$$R_\mu^\nu - \tfrac{1}{2}\delta_\mu^\nu R = 8\pi(\tau_\mu^\nu + t_\mu^\nu)$$

where the left hand side satisfies a well known divergence identity and t_μ^ν is as general as to render the original $g_{\mu\nu}$ a solution. Then the further condition that the total energy-momentum P_μ be conserved can equivalently be imposed on t_μ^ν by requiring that the divergence identity just mentioned leads to the above stated conservation condition. Indeed, the divergence identity gives $(\sqrt{-g})^{-1}\ \partial_\nu(\sqrt{-g}\tau_\mu^\nu) - \tfrac{1}{2}\partial_\mu g_{\alpha\beta}\tau^{\alpha\beta} + D_\nu t_\mu^\nu = 0$, hence the conservation condition becomes /1/

$$\tfrac{1}{2}\partial_\mu g_{\alpha\beta}\tau^{\alpha\beta} = D_\nu t_\mu^\nu$$

which in turn can be used to determine t_μ^ν in conjunction with the rest of the field equations. In other words, under these conditions the field equations will determine the t_μ^ν (up to an irrelevant zero divergence) and guarantee that the total energy-momentum P_μ will become conserved without otherwise altering its magnitude or functional form.

The integral becomes

$$\int \sqrt{-g} D_\nu \tau_\mu^\nu d\Omega = \int \partial_\nu (\sqrt{-g} \tau_\mu^\nu dV_\nu) - \int_{u_0}^{u} \int \sqrt{-g} \sigma du_\mu u^\nu dV_\nu = 0$$

The two integrals are performable, one via Gauss' theorem and the other directly. One has

$$\int \partial_\nu (\sqrt{-g} \tau_\mu^\nu) d\Omega = \int_V \sqrt{-g} \tau_\mu^\nu dV_\nu - \int_{V_0} \sqrt{-g} \tau_\mu^\nu dV_\nu$$

$$\int_{u_0}^{u} \int \sqrt{-g} \sigma du_\mu u^\nu dV_\nu = \int_V \sqrt{-g} \tau_\mu^\nu dV_\nu - \int_{V_0} \sqrt{-g} \tau_\mu^\nu dV_\nu$$

$$Q_\mu - Q_\mu^0 = P_\mu - P_\mu^{(0)} - (P_\mu - P_\mu^{(0)}) = P_\mu - P_\mu - (P_\mu^{(0)} - P_\mu^{(0)}) = 0$$

$$Q_\mu \equiv Q_\mu^{(0)} \equiv 0$$

In other words, in this case the divergence identity turns P_μ into something which is indeed conserved but that something is identically zero. Consequently the Hamiltonian $\tilde{H} = Q_0$ is also identically zero, hence it has no structure, no dynamics . (It is unfortunate that in the past no one seems to have carried out the above integrations explicitly to realize these facts.) Such a Hamiltonian cannot recover the Newtonian dynamics as a limit nor can it predict correctly the 532" per century perturbation effect on the motion of the perihelion. This shows that to correctly predict the effects depending on the Newtonian limit the presence of t_μ^ν is necessary as well as it is sufficient.

This inevitably leads one to the question of why did we believe that general relativity, which has no t_μ^ν in its field equations, predicts correctly all of the effects cited above? It is found by letting /1,4/ $t_\mu^\nu \to \lambda t_\mu^\nu$ that λ has the meaning $P_\mu \to Q_\mu = \lambda P_\mu$ where Q_μ (and therefore also P_μ) is conserved. This gives the impression that there are two Hamiltonian involved, $\tilde{H} = Q_0 = \lambda P_0$ and $H = P_0$. The true Hamiltonian is of course $\tilde{H} = Q_0$ because this is what the field equations with the actual λt_μ^ν produce (without field equations P_μ is not even conserved)*. Since $p_\mu = mu_\mu$, $\tilde{p}_\mu = m' u_\mu$ we can conceive the result as $m' = \lambda m$ where m is the

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* *In other words the true Hamiltonian is $\tilde{H} = \lambda H$ where H is just the $\lambda = 1$ case of it.*

inertial mass m_i. It can be proved /1/ that m' is the "active" (gravity generating) mass, m_a. Since any theory satisfying the geodesic equations of motion satisfies the passive principle of equivalence $m_p = m_i$ we have here a basic difference between the new theory ($\lambda = 1$) which satisfies the "strong" principle of equivalence $m_a = m_p = m_i$ and general relativity ($\lambda = 0$) which satisfies only the passive component. Now what is the effect of this? It turns out that in this respect general relativity is not similar to Newton's theory but to a theory suggested by Hooke a few years before Newton published his own theory in Principia /5/. In Hooke's theory the "passive" principle of equivalence was satisfied but the "active" principle was violated as in above namely $m_a = \lambda m_i$, $\lambda = 0$. As is well known Hooke's theory gave zero "active" mass to planets but a finite active mass to the sun by implicitly assuming the sum has infinite inertial mass. In this way, the sun acted as an immovable (infinite inertial mass) center of force with finite active mass ($m_a = \lambda m_i = 0.\infty \to$ finite) and produced a force field. In this field planets moved according to Kepler's laws (area law because force was central, ellipse because the force was proportional to $1/r^2$, etc.) but the planets had finite inertial mass, hence zero active mass. For this reason planets did not attract each other and there were no planetary perturbations. Writing the above consideration of the "active" mass /6/ as

$$m_a = \lambda m_i$$

$$\lambda = \begin{cases} 0 & \text{Hooke's theory} \leftrightarrow \text{G. Relativity} \\ 1 & \text{Newton's theory} \leftrightarrow \text{New Theory} \end{cases}$$

it becomes clear that, in the low velocity limit, general relativity would reduce to Hooke's theory and not to Newton's theory and would miss the planetary perturbations in the same way as in Hooke's theory. As to the question of why in such a setting general relativity gives the numerical values of the four post-Newtonian effects (namely, the red shift, light-bending, the 43" per century relativistic part of perihelion's motion and radar echo-delay) the answer is that the first three PPN parameters $\alpha = \beta = \gamma = 1$ are independent of λ hence these effects are the same as in the case of $\lambda = 1$. (Note that there are experimental data such as local mass

isotropy and the gravitational phase shift of neutron. These latter type of experiments are found to be consistent with $\lambda = 1$ but not necessarily so in general relativity.) However, it is possible to argue, from the Hooke-Newton analogy, that even the Keplarian orbits (without perturbations) and the four post-Newtonian effects cited are obtainable only by implicitly assuming the sun's inertial mass to be infinite which is contrary to known facts.

DISCUSSION

In this paper we have modified general relativity by adding the gravitational field stress-energy t_μ^ν to the matter stress-energy τ_μ^ν in the field equations consistently with the requirement that the total energy momentum $P_\mu = \Sigma m u_\mu$ is conserved. As such we have a well defined structure which we call the "New Theory". The basic equations of this new theory may be summarized as

$$R_\mu^\nu - \tfrac{1}{2}\delta_\mu^\nu R = 8\pi(\tau_\mu^\nu + t_\mu^\nu) \qquad (1)$$

$$\sigma \frac{du_\mu}{ds} = \tfrac{1}{2}\partial_\mu g_{\alpha\beta}\tau^{\alpha\beta} \qquad (2)$$

$$\Box^2 \phi_\mu^\nu = \tau_\mu^\nu \qquad (3)$$

$$\sigma \frac{du_\mu}{ds} = D_\nu t_\mu^\nu \qquad (4)$$

where Eqs. 2,4 give the conservation condition mentioned in the text. We have here imposed the conservation condition in this way because Eq.4 immediately identifies t_μ^ν as a field stress-energy from the analogy to, say, electromagnetism where t_μ^ν would be the Maxwell stress-enery and $\sigma du_\mu/ds$ the Lorentz force. Eqs.1,2 are self explanatory since they are the equations for $g_{\mu\nu}$ and the equations of motion respectively. Eq.4 where $\Box^2$ is the general d'Alembertian is again very similar to Maxwell theory ϕ_μ^ν being analogous to A_μ and τ_μ^ν analogous to j_μ. These equations may be solved in the Lorentz gauge $\partial_\nu \phi_\mu^\nu = 0$ (equivalent to the Hilbert gauge in first order) and under the correspondence condition

$$\frac{du_\mu}{ds} = -2u_\alpha u^\beta \partial_\mu \phi_\beta^\alpha + \partial_\mu \phi \qquad (5)$$

requiring that, for low velocities the Newtonian equations of motion and for high velocities the bending of light rays are correctly reproduced. A general case satisfying the equations plus the correspondence is the "iteration" solution mentioned in the text. The theory so constructed can be shown to satisfy the strong principle of equivalence $m_g = m_p = m_i$ which is also satisfied by the Newtonian theory.

The choice of the gauge $\partial_\nu \phi_\mu^\nu = 0$ may be sharpened into $\partial_\nu(\sqrt{-g}g^{\mu\nu}) = 0$ which reduces to $\partial_\nu \phi_\mu^\nu = 0$ in all configurations of practical importance. However, with the second form one can immediately prove an important theorem, namely, the coordinates satisfy "sourceless" wave equations $\Box^2 x^\alpha \equiv 0 \; | (\sqrt{-g})^{-1} \; \partial_\nu(\sqrt{-g}g^{\mu\nu}\partial_\mu x^\alpha) = g^{\mu\nu}\partial_\mu\partial_\nu x^\alpha = g^{\mu\nu}\partial_\mu(\delta_\nu^\alpha) \equiv 0 \; |$. The significance of this result is that coordinates cannot become dynamical fields as they do not interact with anything. Although, needless to say, the choice of gauge (or coordinates) is physically irrelevant this informtion is of practical importance because it permits the initial value problem in analogy to Lorentz frames where coordinates likewise satisfy "sourceless" wave equations.

Note carefully that the equations of motion (2) are here written not in the form of geodesics but σ as a factor. This is because without σ the orbits in a multiparticle system will become ambiguous. Even in a two-body problem the masses cannot be dropped from the equations of motion until the foci (in the Newtonian theory the mass center) of the ellipses are determined via the masses. Note carefully that in their correct dynamical form (2) the equations of motion are second order consequences of the field equations because in the Newtonian limit $\frac{1}{2}\partial_\mu g_{\alpha\beta} \to -\partial_\mu\phi$, $\Box^2\phi_\mu^\nu \to -\nabla^2\phi = \sigma$, hence the right hand side is second order as $\sigma du_\mu/ds \to \nabla^2\phi\partial_\mu\phi$. This means that in the treatment of manybody systems the field equations must be examined at least to second order. In the usual general relativity one usually drops σ ahead of time. Then one only has $du_\mu/ds \to -\partial_\mu\phi$ which gives the impression that the equations of motion are first order consequences of the field equations*.

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* *Note that λH gives the same geodesics (Kinematics) for any λ but the values of λ is important because impacts, phase-shifts, etc. (Dynamics) depend on λ. It can be shown that these two aspects are related to the "passive" and "active" principles of equivalence, respectively.*

This impression leads one to the erroneous conclusion that a first order correspondence to the Newtonian theory would recover the full content of the Newtonian dynamics. We believe it is this circumstance that led to an early decision that general relativity would recover the perturbative predictions (such as the 532" per century Newtonian portion of perihelion's motion) without a detailed analysis of the second order situation and its relation to the strong principle of equivalence and/or the conservation laws of energy-momentum. It is unfortunate that in general relativity the Hamiltonian form of the equations of motion is not utilized often enough. If it did one would have noticed that the equations of motion of multiparticle systems will have to be second order because the Hamiltonian contains the interaction energy which involves the products masses.

Finally we may remember that Einstein himself was not very happy with the right hand side of his field equations which we modify. He often compared his equation to a house with the left wing made of "fine marble" and the right wing of "base wood". Our considerations in this paper appear to be strong evidence that he was right in this assessment.

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