SEPARABLE POTENTIALS FOR BOUND STATE CALCULATIONS

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Separable potentials are very useful in the numerical solution of the many- body problem, where they allows to decrease the dimension of the integral equations. So the separable expansion schemes are extensively used in solving both scattering and bound states of a few - body systems. There are a large amount of separable expansion schemes for the two body potential which allows to make the transition operator t satisfying the Lippmann – Schwinger (LS) equation

$$t = V + VG_0 t$$

to be separable, where V is the potential and G_0 is the free Green's Function. It was found that using these schemes a rapid convergence could be obtained in numerical calculations.

In this work we use one of the separable expansion schemes proposed by Ernst, Shakin and Thaler (EST) [1] for the nucleon-^{4}He potential V_n. This scheme enables to provide the potential in a separable form which is equivalent to a given potential.

In the maximally simplified model that proposed in [2] for n interaction, the local part of the potential is identical for all states, simulates the attrac-tion of the neutrons to the-particle nucleons, and has a range in excess of 2 fm. The form of the potential in momentum space is

$$V_{n\alpha}(p,p') = \frac{A}{a}\frac{\sqrt{\pi^3}}{pp'}\left[\exp\left(-\frac{(p-p')^2}{4a^2}\right) + \exp\left(-\frac{(p+p')^2}{4a^2}\right)\right],$$

where a = 0.431 fm^{-1} and A = –50.208 MeV. This potential describes simultaneously the S-wave $n\alpha$ phase shift and low-energy scattering parameters a_0= 2.4641 fm and r_0= 1.385 fm as well as both P-wave phase shifts on average.

Generally, a rank N separable potential has the form

$$V^{sep} = \sum_{i,j=1}^{N} |h_i\rangle \lambda_{ij} \langle h_i|$$

In the EST scheme an object $|h_n\rangle = V|\psi_n\rangle$ is the solution of the LS equation for the angular-momentum eigen-state $|\psi\rangle$

$$|\psi\rangle = |\varphi\rangle + G_0(E)V|\psi\rangle$$

at energy E = E$_n$, where $|\varphi\rangle$ is the incident plane wave and $\{\lambda^{-1}\}_{ij} = \langle\psi_i|V|\psi_j\rangle$. Requiring the equivalence at N energy points, the rank N form of the separable potential is derived by use of the wave functions at the N energy points. The equivalence between separable potential and original one at the N energy points is built in. However the equivalence of the t operators due to each potential off energy shell is only approximate. The form factor of the separable potential for the state with an angular momentum L is defined in momentum space by

$$h_{Li}(p) = \langle p|V_L|\psi_i\rangle \qquad (1)$$

The right-hand side of Eq. (1) is parametrized as follows:

$$h_{Li}(p)=\sum_{k=1}^{N}\frac{C_{Lik}p^{L+2(k-1)}}{(p^2+\beta_{Lik}^2)^{L+k}} \qquad (2)$$

The parameters C_{Lin} and β_{Lin} in (2) were defined from the minimum of the following parameter χ_n^2:

$$\chi_n^2=\frac{\int_0^\infty |\langle p|V_L|\psi_n\rangle-h_{Ln}(p)|^2dp}{\int_0^\infty |\langle p|V_L|\psi_n\rangle|^2}.$$

The parameters C_L ,β_L and λ for the N = 4 rank separable potential in all partial waves are (4x4) dimension matrices. We displayed their matrix elements below for the S -wave case when L = 0 at the energy points of 5,10, 15, 20 MeV:

$$C_0[fm^0]=\begin{pmatrix} -20{:}938201 & 140.649946 & -291.958566 & 172.326604 \\ -25.499041 & 149.685929 & -316.445150 & 192.343806 \\ -1.954185 & -60.227815 & -12.973544 & 74.9086530 \\ -3.419409 & -535.867632 & 429.711980 & 109.503317 \end{pmatrix}$$

$$\beta_0[fm^{-1}]=\begin{pmatrix} 1.221392 & 1.794451 & 1.367044 & 1.061224 \\ 1.529969 & 1.917307 & 1.461015 & 1.141552 \\ -0.466188 & 1.452132 & 0.943265 & 0.945710 \\ 0.721409 & 3.158677 & 2.778636 & 1.266961 \end{pmatrix}$$

$$\lambda[fm^{-1}]=\begin{pmatrix} 0.0507482 & -0.0769347 & -0.0020682 & -0.0331138 \\ -0.0769347 & 0.2342660 & -0.1516350 & -0.0177153 \\ -0.0020682 & -0.1516350 & 0.5019120 & -0.3815200 \\ -0.0331138 & -0.0177153 & -0.3815200 & 0.5221050 \end{pmatrix}$$

Here inside the square brackets the units of the C_L, β_L and λ are showed. By use of the orthogonal pseudo potential method we constructed a new nα- potential which takes into account the forbidden state in n-α system. Three - body bound state calculation of the ^{6}He nuclei using this separable potential in the (^{4}He + n + n) - channel is under way.

References

1. D. J. Ernst, C. M. Shakin, R. M. Thaler, Phys. Rev. C8, 507A (1973)
2. B. E. Grinyuk, I. V. Simenog, Phys. At. Nucl. V.72, No.1, pp.6-19 (2009)