# PHASE SHIFT ANALYSIS OF $p^{12}C$ SCATTERING AT ENERGIES 0.2-1.1 MeV

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## ABSTACT

The experimental data of cross-sections of nuclear reactions and their analysis within the frames of various theoretical models are the major source of information about nuclear structure, nature and mechanism of nuclear-nuclear (cluster-cluster) interaction. Researches in nuclear astrophysics are more complicated since in many cases only theoretical predictions can supply deficient experimental information on characteristics of nuclear reactions. This difficulty results from low energy of interraction in substance matter of stars which equals from tenth to tens keV and thus, there is no possibility of direct cross-section measurments which are required in astrophysical caculations of nuclear reactions. Generally, cross-section measurments are carried out at higher energies (about hundreds of keV) and then they are extrapolated into the energy range representing interest for nuclear astrophysics.

## 1.INTRODUCTION

The reliability of such prosidure depends, first of all, on experimental measurement accuracy of extrapolated characteristics. Thus improvement of measurement accuracy and widening class of measured reactions which take place with the same nuclei at defined energy can decrease the ambiguity of theoretical analysis and the uncertainty of such extrapolations. To that end the detailed measurements of differential cross-sections of elastic $p^{12}C$ scattering at energies from 0.2 up to 1.1 MeV (center-of-mass system) within the range of $10^0$-$170^0$ with 10% errors was done in works [1, 2].

The standard phase shift analysis was done and the potential of S-state of $p^{12}C$ system was reconstructed in this paper on the basis of measurements. Cross-section of elastic scattering of nuclear particles can be represented as in [3]

$$\frac{d\sigma(\theta)}{d\Omega} = |A(\theta)|^2 + |B(\theta)|^2 , \tag{1}$$

$$A(\theta) = f_c(\theta) + \frac{1}{2ik}\sum_{L=0}^{\infty}\{(L+1)S_L^+ + LS_L^- - (2L+1)\}\exp(2i\sigma_L)P_L(Cos\theta),$$

$$B(\theta) = \frac{1}{2ik}\sum_{L=0}^{\infty}(S_L^+ - S_L^-)\exp(2i\sigma_L)P_L^1(Cos\theta) .$$

Here, $S_L^{\pm} = \eta_L^{\pm}\exp(2i\delta_L^{\pm})$ are scattering matrixes, $\eta^{\pm}_L$ - inelasticity parameters, "±" signs correspond to total moments of the system ($J = L \pm 1/2$), $f_c$ - Coulomb amplitude, $\eta$ - Coulomb parameter, $\mu$ - reduced mass, k - wave number of relative motion of particles, $k^2 = 2\mu E/\hbar^2$ were E is the energy of colliding particles in the center-of-mass system.

Using experimental data of differential cross-sections of scattering it is possible to find some set of phase shifts $\delta_{S,L}^J$ which is able to reproduce behavior of these cross-sections with certain accuracy. Quality of description of experimental data on the basis of some theoretical function (functional (1) of several variables) can be estimated by the $\chi^2$ method which is written as

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_i^t(\theta) - \sigma_i^e(\theta)}{\Delta\sigma_i^e(\theta)}\right]^2 = \frac{1}{N}\sum_{i=1}^{N}\chi_i^2 ,$$

where $\sigma^e$ and $\sigma^t$ - experimental and theoretical, i.e. calculated at some defined values of phase shifts $\delta_{S,L}^J$ of scattering, cross-sections of elastic scattering of nuclear particles for i-angle of scattering, $\Delta\sigma^e$ - error of experimental cross-sections at these angles, N - number of measurements.

The less $\chi^2$ value, the better description of experimental data on the basis of a chosen theoretical conception i.e. a certain phase shift set. Usually calculation results can be considered quite satisfactory if total $\chi^2$ is about one i.e. deviation of experimental and theoretical values from each other is in average equal to the value of experimental error. The description is good when every partial $\chi^2_i$ for each scattering angle is lower than one, hence the average $\chi^2$ will always be lower than one.

Expressions (1) are expansions of some functional $d\sigma(\theta)/d\Omega$ to the numerical series and it is necessary to find such variation parameters of expansion $\eta_L$ and $\delta_L$ which best describe experimental data. Since the expressions given above (1) are exact, then as L approaches infinity the value of $\chi^2$ must vanish to zero. This criteria is used for choosing a certain set of phase shifts ensuring the minimum of $\chi^2$ which could possibly be the global minimum of a multiparameter variation problem.

For searching nuclear phase shifts of scattering using experimental cross-sections, the procedure of minimization of functional $\chi^2$ as a function of 2L variables, each of which is a phase shift $\delta_L$ of a certain partial wave of scattering and inelasticity $\eta_L$ in this wave, was carried out. For solving this problem the minimum of $\chi^2$ is searched for within some limited range of values for such variables. Particularly, $\eta_L$ can take on the values from 0 to 1 and phase shifts $\delta_L$ are usually in the interval from 0 to 180 degrees. But it is possible to find a lot of local minima of $\chi^2$ with the value of about one in this range. Choosing the smallest of them allows hoping that this minimum will correspond to the global minimum which is a solution of such variation problem. Then, the value of this minimum should decrease more or less smoothly as the number of partial waves increases. Sets of scattering phase sifts not leading to such a decrease were not taken into consideration as parameter variants resulting in local minima.

Earlier, phase shift analysis of excitation functions of $p^{12}C$ scattering, measured in [4] at energies in the range of 400-1300 keV (l.s.-laboratory system) and angles from $106^0$ up to $169^0$, was done in work [5] where it was found that S-phase must be in the range of $153^0$-$154^0$ at the energy $E_{lab}$ = 900 keV. For the same experimental data we have received $152.7^0$. Cross-sections were extracted from excitation functions at energies 866-900 keV. Results of our calculations $\sigma_t$ in comparison with experimental data $\sigma_e$ are given in Table 1. Partial $\chi^2_i$ for each point with 10% errors in experimental cross-sections are given in the last column of the table and the value of 0.11 is received for the average of $\chi^2$.

Values $155^0$-$157^0$ of S-phase shift are found in work at the energy 751 keV (l.s.). We took the cross-section data from excitation functions at energies in the range of 749-754 keV and obtained the value $156.8^0$ for S-phase shift at the average $\chi^2$ = 0.30.

Table 1. Comparison of theoretical and experimental cross-sections of $p^{12}C$ elastic scattering at energy 900 keV.

| $\theta$, deg. | $\sigma_e$, mb. | $\sigma_t$, mb. | $\chi^2_i$ |
|---|---|---|---|
| 106 | 341 | 341.5 | 1.90E-04 |
| 127 | 280 | 282.1 | 5.76E-03 |
| 148 | 241 | 251.2 | 1.80E-01 |
| 169 | 250 | 237.5 | 2.50E-01 |

All these results are in a good agreement with each other and the phase shift analysis of new experimental data of differential cross-section of $p^{12}C$ scattering was carried out using our program at energies in the range of 230-1200 keV (l.s.). The results of our phase shift analysis are shown in Fig.1 (points - phase shift analysis taking into account only S-waves, open blocks - phase shift analysis with S and P waves) and in Fig.2 (points and blocks) and are given in Table 2 in comparison with the values of work which are shown with the solid line in the figure.

Figure 3 shows the differential cross-sections in the resonance region at 461(3) keV (l.s.), calculation results of these cross-sections on the basis of Rutherford formula - (dot line), cross-sections on the basis of our phase shift analysis which takes into account S-phase shift only - (solid line). The cross-sections for S-phase shift allowing for S- and P-waves are shown in the figure by the dash line.

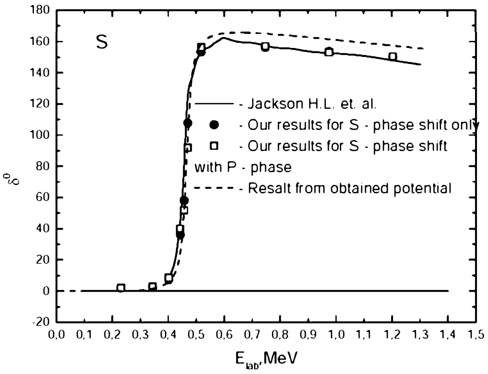


Fig. 1. S- phase shift of elastic $p^{12}C$ scattering. Points -results of phase shift analysis for S-phase shift taking into account S-wave only, open blocks - results of phase shift analysis for S-phase shift taking into account S- and P-waves, solid line - results of work, dash line - results calculated with the potential found here.

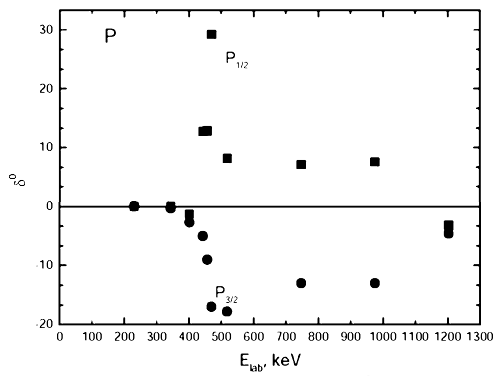


Fig. 2. P-phase shift of elastic $p^{12}C$ scattering. Points and blocks are the results of phase shift analysis for P-phase shift taking into account S- and P-waves.

Table 2. Results of phase shift of elastic $p^{12}C$ scattering at low energies taking into account S-wave only.

| $E_{cm}$, keV | $\delta_S$, deg. | $\chi^2$ |
|---|---|---|
| 213 | 2.0 | 1.35 |
| 317 | 2.5 | 0.31 |
| 371 | 7.2 | 0.51 |
| 409 | 36.2 | 0.98 |
| 422 | 58.0 | 3.75 |
| 434 | 107.8 | 0.78 |
| 478 | 153.3 | 2.56 |
| 689 | 156.3 | 2.79 |
| 900 | 153.6 | 2.55 |
| 1110 | 149.9 | 1.77 |

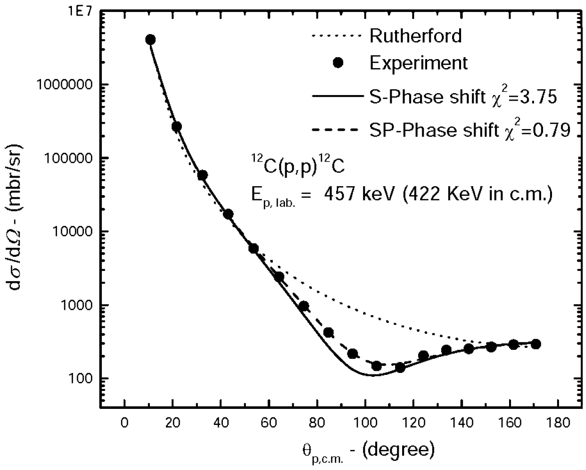


Fig. 3. Differential cross-sections of $p^{12}C$ scattering. Solid line is a phase shift analysis which takes into account S-wave only, dot line - Rutherford scattering, dash line - phase shift analysis where S- and P-waves are taken into account, points - experiment data.

One can see from the figures that it is impossible to describe the cross-section in the resonance region on the basis of S-wave only. In this case, the P-wave shown in Fig.2 begins to play a noticeable part, correction for this effect (dash line in Fig.3) gives the obvious progress in experiment description. At the energy 457 keV (l.s.) the value of $\chi^2$.can be improved from 3.7 to 0.8 (Fig.3) by taking into account P-wave, what considerably affects the quality of description of differential cross-sections.

At the low energies $P_{1/2}$-phase shift lies above $P_{3/2}$ but at the energy about 1.2 MeV they intersect and then $P_{3/2}$-phase shift lies above in the region of negative angles [6,7]. The value of S-phase shift practically doesn't change when P-wave is taken into account (see Fig.1 - open blocks). Allowing for D-wave in the phase shift analysis leads to the value 1-1.5 degrees in the resonance region and practically doesn't affect behavior of calculated differential cross-sections.

The S-phase shift showed in Fig.1 can be described on the basis of simple Gaussian potential with pointlike Coulomb component:

$$V(r) = V_0 \exp(-\alpha r^2) + V_{cul}$$

were $V_0 = 29.87$ MeV, $\alpha = 0.22$ $fm^{-2}$. The constant $\hbar^2 / m_0 = 41.80159$ was used in calculations, $m_0$ - atomic mass unit [8].

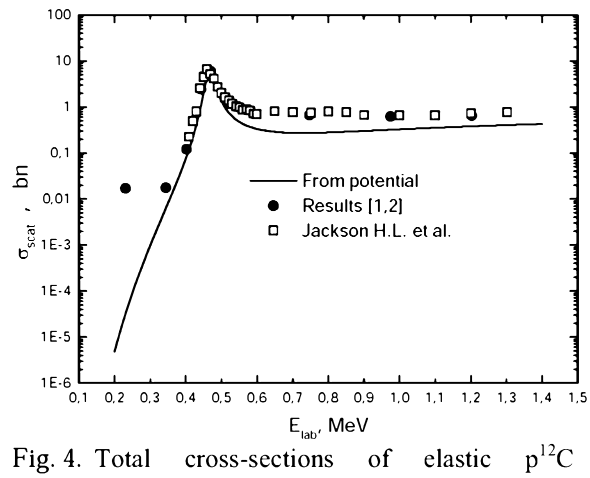


Fig. 4. Total cross-sections of elastic $p^{12}C$ scattering. Solid line - calculation results on the basis of found potential, points - total cross-sections found with the phase shifts from works, blocks - the same kind of thing for works.

The calculation results of S-phase shift with such potential are shown in Fig.1 by the dash line. The phase shift reaches the value of $90^0$ at the energy 467 keV what is in a good agreement with the experimental resonance value 461(3) keV (l.s.) [9].

Total cross-sections of elastic scattering are shown by points in Fig.4. They are calculated on the basis of S-phase shifts of scattering extracted from experimental differential cross-sections. Here, the solid line shows the total cross-sections found on the basis of phase shifts calculated with abovementioned potential. Open blocks are the total cross-sections obtained from the phase shift analysis of work.

As one can see in Fig.4 the values of total cross-sections in the energy range 200-300 keV (l.s.) reach a plateau and hence the same will be with S-phase shift of scattering. Presently, it is not clear if it results from experimental inaccuracies, unforeseen errors of phase shift analysis or it really exists in this range of energies.

New measurements of angular distribution of $p^{12}C$ scattering in the energy range from 100-150 to 300-350 keV with energy step about 50 keV or of excitation functions at different angles are needed for clearing it up.

## 2. REFERENCES


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