PHASE SHIFT ANALYSIS OF THE ELASTIC $p^{12}C$ SCATTERING DIFFERENTIAL CROSS SECTIONS AT ASTROPHYSICAL ENERGIES

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ABSTACT

Using the earlier obtained experimental data on the $p^{12}C$ elastic scattering at the energies $E_{c.m.}$ = 213, 317, 371, 409, 422, 434, 478, 689, 900, 1110 keV and in the range of angles $\theta_{lab.} = 10^0 - 170^0$ the phase shift analysis has been made and the interaction potential of the $p^{12}C$ system in S- state has been constructed. The standard optical model analysis of the angular distributions has also been carried out.

1. INTRODUCTION

Experimental data on nuclear processes cross sections and their analysis within the framework of various theoretical models are the basic source of the information about nuclear structure, properties and mechanisms of nucleus-nucleus (cluster-cluster) interactions. In the nuclear astrophysics it is often the case that only theoretical predictions can give the missing information about the process characteristics. This happens because the interaction energy of substance in stars is relatively small (from tenths to hundreds keV) and so there is no opportunity to measure directly nuclear process cross sections, which are required for astrophysical calculations. Usually measurements of cross sections are made at higher energies and then, with the use of theoretical methods, they are extrapolated into the range of energies of interest to astrophysics and thermonuclear synthesis. Reliability of such procedure is determined by quality of theoretical models and accuracy of measurement of extrapolated characteristics. Therefore increase in the accuracy of measurement and expansion of the class of measured processes proceeding on same nuclei at a set energy can reduce ambiguity of the theoretical analysis and decrease the extrapolation uncertainty.

Toward this end, previously, at low energies and in a wide range of scattering angles, the measurement of differential cross sections (angular distributions and excitation functions) of the $p^{12}C$ elastic scattering had been made at the electrostatic tandem accelerator UKP-2-1 in the Institute of Nuclear Physics in Almaty [1,2]. In this work the standard optical fitting of potentials is carried out, the phase shift analysis of the experimental data from [1,2] is performed and comparison of this analysis with other known results is given.

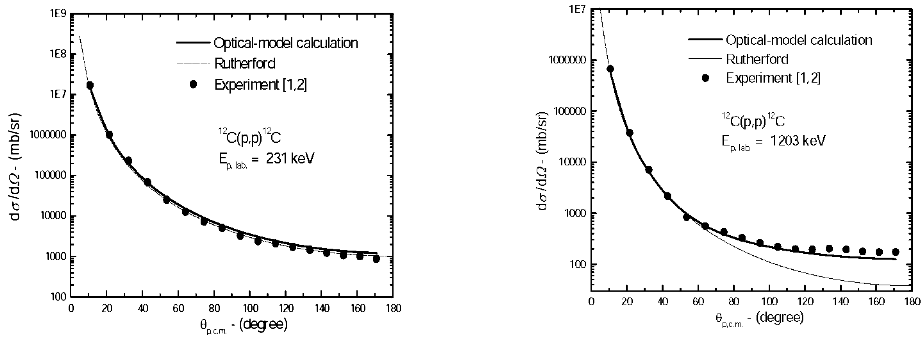
2. OPTICAL MODEL

On the basis of the earlier obtained experimental data [1,2] the parameters of the optical potentials' (see table 1) have been fitted [2]. These parameters differ little from the similar results obtained in the work [3] and have no energy dependence since a relatively narrow energy range was considered.

Satisfactory quality of the differential cross section description at energies $E_{c.m.}$ = 213 and 1110 keV is shown in fig. 1a and 1b in comparison with the cross sections of the Rutherford scattering. The acceptable agreement between the calculation and experiment is observed for all energies examined in works [1,2], except for the energy 457 keV (or 422 keV in the center-of-mass system), which practically falls into the region of resonance for the $^{12}C(p,\gamma)^{13}N$ reaction at $E_{p,\,lab.}$ = 461 (3) keV. At this energy a resonant part of the cross sections, which is not described by optical model, introduces a substantial contribution to the cross sections of the elastic scattering.

Table 1. Optical-model parameters

V_R, MeV	r_R, fm	a_R, fm	W_D, MeV	r_{WD}, fm	a_{WD}, fm	V_{SO}, MeV	r_{SO}, fm	a_{SO}, MeV
53	1.4	0.75	2	1.25	0.4	7	1.4	0.5



a

b

Figure 1. Differential cross sections of the $p^{12}C$ elastic scattering

3. PHASE SHIFT ANALYSIS METHODS

While examining scattering in the particle system with a total spin 1/2, i.e. where one of the particles has spin 0, and the second has spin 1/2, it is necessary to take into account spin - orbital split of phase shifts. This sort of scattering takes place in the nuclear systems such as N^4He, $^3H^4He$, $p^{12}C$ etc. The differential cross section of the elastic scattering of nuclear particles is represented as [4]

$$\frac{d\sigma(\theta)}{d\Omega} = |A(\theta)|^2 + |B(\theta)|^2 , \tag{1}$$

where

$$A(\theta) = f_c(\theta) + \frac{1}{2ik}\sum_{L=0}^{\infty}\{(L+1)S_L^+ + LS_L^- - (2L+1)\}\exp(2i\sigma_L)P_L(Cos\theta) , \tag{2}$$

$$B(\theta) = \frac{1}{2ik}\sum_{L=0}^{\infty}(S_L^+ - S_L^-)\exp(2i\sigma_L)P_L^1(Cos\theta) .$$

Here $S_L^{\pm} = \eta_L^{\pm}\exp(2i\delta_L^{\pm})$ is the scattering matrix, $\eta^{\pm}_L$ are the parameters of inelasticity, and "±" correspond to the total moment of the system J = L ± 1/2, f_c is the Coulomb amplitude presented in the form of

$$f_c(\theta) = -\left(\frac{\eta}{2kSin^2(\theta/2)}\right)\exp\{i\eta\ln[Sin^{-2}(\theta/2)] + 2i\sigma_0\} ,$$

$P_n^m(x)$ are the associated Legendre polynomials or functions

$$P_n^m(x) = (1-x^2)^{m/2}\frac{d^m P_n(x)}{dx^m} ,$$

which can be calculated by recurrent formulas

$$P_{L+1}^1(x) = \frac{(2L+1)x}{L}P_L^1(x) - \frac{L+1}{L}P_{L-1}^1(x)$$

with initial values

$$P_0^1(x) = 0 , \quad P_1^1(x) = (1-x^2)^{1/2} , \quad P_2^1(x) = 3xP_1^1(x) ,$$

η is the Coulomb parameter, μ is the reduced mass, k is the wave number of relative movement of the particles $k^2 = 2\mu E/\hbar^2$, E is the energy of the colliding particles in center-of-mass system.

It is also possible to express the polarization in the elastic scattering of such particles [4] in terms of quantities A and B (2)

$$P(\theta) = \frac{2\,\mathrm{Im}(AB^*)}{|A|^2 + |B|^2} \quad . \tag{3}$$

By unfolding the expression (2) we obtain the following for B:

$$\mathrm{Re}\,B = \frac{1}{2k}\sum_{L=0}^{\infty}[a Sin(2\sigma_L) + b Cos(2\sigma_L)]P_L^1(x) \quad ,$$

$$\mathrm{Im}\,B = \frac{1}{2k}\sum_{L=0}^{\infty}[b Sin(2\sigma_L) - a Cos(2\sigma_L)]P_L^1(x)\,, \tag{4}$$

where

$$a = \eta_L^+ Cos(2\delta_L^+) - \eta_L^- Cos(2\delta_L^-)$$
$$b = \eta_L^+ Sin(2\delta_L^+) - \eta_L^- Sin(2\delta_L^-) \quad .$$

In a similar way, it is possible to find the following expressions for amplitude A [5]

$$\mathrm{Re}\,A = \mathrm{Re}\,f_c + \frac{1}{2k}\sum_{L=0}^{\infty}[c Sin(2\sigma_L) + d Cos(2\sigma_L)]P_L(x) \quad ,$$

$$\mathrm{Im}\,A = \mathrm{Im}\,f_c + \frac{1}{2k}\sum_{L=0}^{\infty}[d Sin(2\sigma_L) - c Cos(2\sigma_L)]P_L(x)\,, \tag{5}$$

where

$$c = (L+1)\eta_L^+ Cos(2\delta_L^+) + L\eta_L^- Cos(2\delta_L^-) - (2L+1)$$
$$d = (L+1)\eta_L^+ Sin(2\delta_L^+) + L\eta_L^- Sin(2\delta_L^-) \quad .$$

For calculation of polarization it is possible to use the following relations

B = ReB + iImB = C + iD , A = ReA + iImA = G + iF,

AB^* = CG + FD + i(FC - GD) , Im(AB^*) = FC - GD = ReB ImA - ReA ImB,

which are directly applied to the numerical computer calculations.

For the total cross section of reactions there is an expression similar to that for a system with zero spin [4]

$$\sigma_r = \frac{\pi}{k^2}\sum_L\left[(L+1)\left(1 - \left|S_L^+\right|^2\right) + L\left(1 - \left|S_L^-\right|^2\right)\right] \quad , \tag{6}$$

or

$$\sigma_r = \frac{\pi}{k^2}\sum_L\left\{(L+1)[1 - (\eta_L^+)^2] + L[1 - (\eta_L^-)^2]\right\} \quad .$$

The total cross section of the elastic scattering is given by

$$\sigma_s = \frac{\pi}{k^2}\sum_L\left[(L+1)\left|1 - S_L^+\right|^2 + L\left|1 - S_L^-\right|^2\right] \tag{7}$$

or

$$\sigma_s = \frac{4\pi}{k^2}\sum_L \left\{(L+1)[\eta^+ Sin\delta^+]^2 + L[\eta^- Sin\delta^-]^2\right\}$$

Knowing the experimental differential cross sections of the elastic scattering, it is possible to find some set of phase shifts $\delta^J_{S,L}$ which can describe behavior of these differential cross sections more or less accurately. The quality of the experimental data description on the basis of some theoretical function (functional of several variables) can be estimated using the χ^2 - method, which is represented as

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma^t_i(\theta) - \sigma^e_i(\theta)}{\Delta\sigma^e_i(\theta)}\right]^2 = \frac{1}{N}\sum_{i=1}^{N}\chi^2_i \quad ,$$

where σ^e and σ^t are the experimental and theoretical(i.e. calculated by using some given values of the scattering phase shifts $\delta^J_{S,L}$) differential cross sections of the elastic scattering of nuclear particles for i - th scattering angle, $\Delta\sigma^e$ is the error of the experimental differential cross sections for these angles, N – the number of measurements.

The less the value χ^2 is, the better is the description of the experimental data in terms of the selected theoretical representation. Usually the results of calculations can be considered as wholly satisfactory if χ^2 is about 1, i.e. the deviation of the calculated vales from the experimental ones is in average equal to the value of the experimental errors. The description can be considered as good if every partial χ^2 for each scattering angle is less than 1, and thus, the average χ^2 is always less than 1.

The expressions (1,2) are in essence the expansion of some function $d\sigma(\theta)/d\Omega$ into numerical series, and it is necessary to find such variational parameters of η_L and δ_L expansions, that describe this function the best. Since the introduced expressions (1,2) are precise, then the value χ^2 must vanish to zero as L approaches infinity. This criterion was used for selection of a fixed set of phase shifts, which give the minimum of χ^2 and which could possibly play the role of the global minimum of the multiparameter variational problem.

To find the nuclear phase shifts of the scattering with the use of the experimental differential cross sections, the procedure of minimization of the functional χ^2, which is a function of 2L variables - the phase shift δ_L of a certain partial wave of the elastic scattering and the inelasticity η_L in this wave, was carried out. To solve this problem it is necessary to find the minimum of χ^2 in some finite range of values of these variables. In particular, the inelasticity η_L can take only the values from 0 up to 1, and the phase shifts δ_L are usually in the range of 0^0-180^0. But even in this range it is possible to find a great number of local minima of χ^2 with the values about 1. Choosing the least of them, one may hope that it will correspond to the global minimum, that is, the solution of the variational problem.

The source code of the computer program for calculation of the total and differential cross sections of elastic scattering of particles with spin 1/2 and for phase shift analysis, tested with the p^4He system, is given in work [6].

Only one variant of the control calculation by using this program is put in here. This calculation has been performed for the p^4He elastic scattering with the data from work [7], where the phase shift analysis for energy $E_{p,\ lab.}$ = 9.89 MeV is made. In this analysis positive D phase shifts of the scattering and average χ^2=0.60 are obtained. For our analysis 22 points from the differential cross sections at energy 9.954 MeV from work [8] (in work [7] it is not denoted exactly which 22 points were taken out of 24 ones presented in work [8]) and some polarization points from works [7,9] were used. It is likely that in the latter the data at angles 46.5^0, 55.9^0, 56.2^0, 73.5^0, 89.7^0, 99.8^0, 114.3^0, 128.3^0 and energies $E_{p,\ lab.}$ = 9.89, 9.84 and 9.82 MeV was used.

The phase shifts from work [7] are shown in tab. 2, and computations of the average χ^2_σ with the use of the program [5,6], taking into account 24 points of the cross sections from [8] (at energy $E_{p,\ lab.}$ = 9.954 MeV) and these phase shifts, give the value of 0.586, as shown in tab. 3, with experimental errors being 2%.
Taking the experimental data on polarizations from works [7,8] at energy range $E_{p,\ lab.}$ = 9.82-9.89 MeV and eight scattering angles with phase shifts from [7], it is possible to obtain the results shown in tab. 4 for χ^2_p = 0.589 (the angles are given in degrees, the polarizations are in percentage, and the energy is assigned to be equal to 9.954 MeV, as usually).

Averaging χ^2 over all points (24+10=34), the results for which are mentioned above, i.e. using the expression

$$\chi^2 = \frac{1}{(N_\sigma + N_P)} \left\{ \sum_{i=1}^{N} \left[\frac{\sigma_i^t - \sigma_i^e}{\Delta\sigma_i^e} \right]^2 + \sum_{i=1}^{N} \left[\frac{P_i^t - P_i^e}{\Delta P_i^e} \right]^2 \right\} = \frac{1}{(N_\sigma + N_P)} \{\chi^2_\sigma + \chi^2_P\}$$

Table 2. Phase shifts of scattering from work [7]

E, MeV	S_0, deg.	$P_{3/2}$, deg.	$P_{1/2}$, deg.	$D_{5/2}$, deg.	$D_{3/2}$, deg.
9.954	$119,3^{+2.0}_{-1.8}$	$112,4^{+3.5}_{-5.2}$	$65,7^{+2.7}_{-3.2}$	$5,3^{+1.6}_{-2.5}$	$3,7^{+1.6}_{-2.8}$

Table 3. Experimental and theoretical differential cross sections of the p^{4}He scattering

θ^0	σ_e, mb.	σ_t, mb.	χ^2_i	θ^0	σ_e, mb.	σ_t, mb.	χ^2_i
25.10	371.00	366.85	0.31	109.90	21.00	20.70	0.51
30.89	339.00	331.54	1.21	120.60	23.00	22.59	0.79
35.07	305.00	308.40	0.31	122.80	24.50	24.19	0.40
49.03	232.00	230.61	0.09	130.13	31.90	31.91	0.00
54.70	205.00	199.10	2.07	130.90	33.20	32.90	0.21
60.00	176.00	170.56	2.39	134.87	37.80	38.44	0.71
70.10	124.00	120.59	1.89	140.80	47.30	47.69	0.17
80.00	82.00	79.77	1.85	145.00	54.00	54.62	0.33
90.00	49.20	48.82	0.15	149.40	61.60	61.88	0.05
94.07	39.10	39.44	0.19	154.90	70.60	70.54	0.00
102.17	26.20	26.39	0.13	160.00	78.40	77.71	0.19
106.90	22.00	22.15	0.12	164.40	83.00	82.94	0.00

Table 4. Polarizations in the p^{4}He scattering. Here P_e are the experimental polarizations, ΔP_e are the experimental errors for polarizations and P_t are the calculated polarizations

θ^0	P_e, %	ΔP_e	P_t, %	χ^2_i
46.50	-32.30	2.10	-33.11	0.15
55.90	-41.30	2.20	-42.50	0.30
56.20	-44.40	0.90	-42.81	3.11
73.50	-62.60	3.00	-62.84	0.01
73.50	-64.80	1.90	-62.84	1.06
89.70	-76.10	3.60	-76.33	0.01
89.70	-75.50	2.40	-76.33	0.12
99.80	-59.30	2.50	-58.55	0.09
114.30	48.20	3.20	51.03	0.78
128.30	99.40	3.30	97.66	0.28

gives the value $\chi^2 = 0.5875 \approx 0.59$, in full accordance with the results of work [7], where the value $\chi^2 = 0.60$ is presented. Here N_σ and N_P are the number of the data on the differential cross sections (24 points) and polarizations (10 points), σ^e, P^e, σ^t, P^t are the experimental and theoretical values of the differential cross sections and polarizations, $\Delta\sigma$ and ΔP are their errors.

Additional minimization of χ^2 with the use of the program [5,6] results in $\chi^2_\sigma = 0.576$ for the differential cross sections , $\chi^2_p = 0.561$ for the polarizations and the total $\chi^2 = 0.572$ for the following values of phase shifts $S_0 = 119.01$, $P_{3/2} = 112.25$, $P_{1/2} = 65.39$, $D_{5/2} = 5.24$, $D_{3/2} = 3.63$, what lies completely within the error band presented in work [7] and shown in tab. 2.

4. PHASE SHIFT ANALYSIS IN THE $p^{12}C$ SYSTEM

Earlier, phase shift analysis of the excitation functions, measured in [10] within the energy range $E_{p,\,lab.} = 400 - 1300$ keV and angles from 106^0 to 169^0,was made in work [11]. Fig. 2 shows the excitation function obtained in work [11] for angle $\theta_{c.m.} = 169.2^0$ and the results of works [1,2] at $\theta_{c.m.} = 170^0$. It also shows that both results are in a good agreement at all coincident energies within the range $E_{c.m.} = 350\text{-}1000$ keV.

In work [11] it was obtained that, for example, at $E_{p,\,lab.} = 900$ keV, S – phase shift must be in the range of $153^0\text{-}154^0$. Using the same experimental data in this work we obtained $\delta_S = 152.7^0$. The differential cross section of the elastic scattering was taken from the excitation functions [11] at energies $E_{p,\,lab.} = 866\text{-}900$ keV. Table 5 shows the results of our calculations for σ_t versus the experimental data σ_e. The partial χ^2_i of every point with 10% errors in the experimental differential cross sections are given in the last column of tab. 5. For the average χ^2 the value of 0.11 was obtained.

In work [11] for energy $E_{p,\,lab.} = 751$ keV, S – phase shift, $\delta_S = 155^0\text{-}157^0$ was obtained. The results obtained in this work for energy $E_{p,\,lab.} = 750$ keV are shown in tab. 6. The data on the differential cross sections was taken from the excitation functions for the energy range $E_{p,\,lab.} = 749\text{-}754$ keV, while for the S – phase shift we found $\delta_S = 156.8^0$ with the average $\chi^2 = 0.30$.

All these results are in good agreement with each other, and by means of the program [5,6] the phase shift analysis of new experimental data from [1,2] on differential cross sections of the $p^{12}C$ elastic scattering for the energy range $E_{p,\,lab.} = 230\text{-}1200$ keV was made.

The results of this phase shift analysis are shown with points in fig. 3 and tab. 7 in comparison with the data of work [11], which are depicted in the figure with a continuous line. It is seen from fig. 3 that the results of this phase shift analysis conform to the data of work [11]. In the present analysis the value χ^2 was calculated at 10% experimental errors of the differential cross sections.

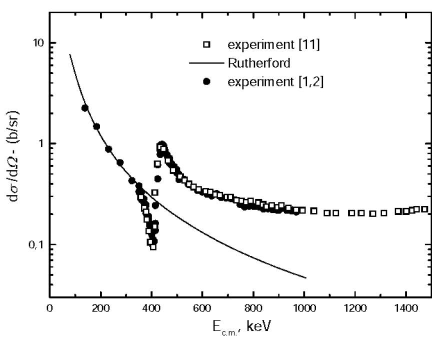


Figure 2. Excitation functions of the $p^{12}C$ elastic scattering for $\theta_{c.m.} = 170^0$ [1,2] and 169.2^0 [11]

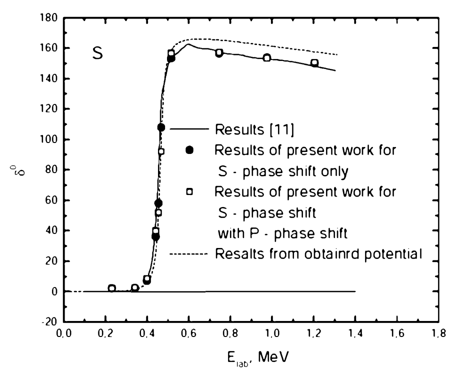


Figure 3. S – phase shift of the $p^{12}C$ scattering at low energies

Table 5. Comparison between the theoretical and experimental differential cross sections of the $p^{12}C$ elastic scattering at energy $E_{p,\,lab.} = 900$ keV

θ^0	σ_e, mb.	σ_t, mb.	χ^2_i
106	341	341.5	1.90E-04
127	280	282.1	5.76E-03
148	241	251.2	1.80E-01
169	250	237.5	2.50E-01

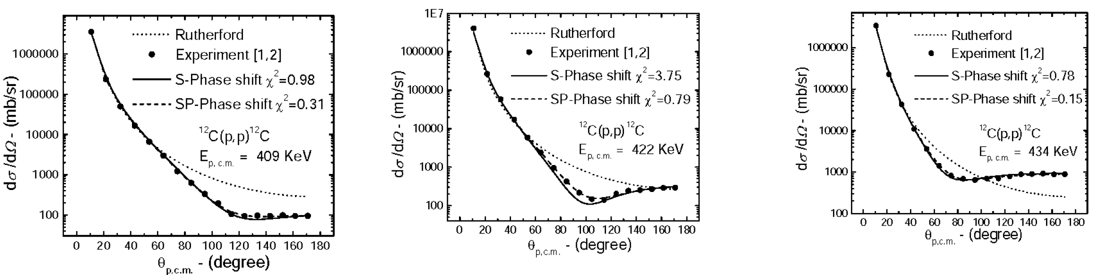
Table 6. Comparison between the theoretical and experimental differential cross Sections of the $p^{12}C$ elastic scattering at energy $E_{p,\,lab.}$ = 750 keV

θ^0	σ_e, mb.	σ_t, mb.	χ^2_i
106	428	428.3	3.44E-05
127	334	342.8	6.91E-02
148	282	299.1	3.66E-01
169	307	279.9	7.82E-01

Table 7. Results of the phase shift analysis of the $p^{12}C$ elastic scattering at low energies

$E_{c.m.}$, keV	S, deg.	χ^2
213	2.0	1.35
317	2.5	0.31
371	7.2	0.51
409	36.2	0.98
422	58.0	3.75
434	107.8	0.78
478	153.3	2.56
689	156.3	2.79
900	153.6	2.55
1110	149.9	1.77

The experimental differential cross sections at energies near the resonance at $E_{p,\,lab.}$ = 461 (3) keV (solid circles), the results of these cross sections calculation on the basis of the Rutherford formula (dot curve) and this phase shift analysis (continuous line), which take into account only S-phase shift, are shown in fig. 4 a, b, c. The results of the phase shift analysis for S-phase shift allowing for S- and P-waves are shown with a dashed line.



a b c

Figure 4. Differential cross sections of the $p^{12}C$ elastic scattering

From the figures one can see, that at the region near the resonance it is impossible to describe the differential cross sections using only S – phase shift. P-wave, shown in fig. 5, starts to play a noticeable role, and its inclusion (dashed line in fig. 4a) considerably improves the description of the experimental data. In particular, at energy $E_{p,\,lab.}$ = 457 keV, taking P-wave into account changes the value of χ^2 from 3.7 to 0.8 (fig. 4b), what has a significant effect on the quality of the differential cross sections description.

At low energies $P_{1/2}$ phase shift goes above $P_{3/2}$, but at energy about 1.2 MeV they intersect and further $P_{3/2}$ goes above in the negative range of angles [12,13]. The value of S – phase shift allowing for P - wave practically does not change and its form is shown in fig. 3 with open squares. At some energies the points and squares completely cover each other.

Taking D-wave into account in the phase shift analysis leads to its phase shift value about 1-1.5 degree in the range of energies near resonance and has almost no effect on the behavior of the calculated differential cross sections.

5. POTENTIAL DESCRIPTION OF THE $p^{12}C$ ELASTIC SCATTERING

S – phase shift shown in fig. 3 can be described using the simple Gaussian potential with the point Coulomb potential of the form

$$V(r) = V_0 \exp(-\alpha r^2) + V_{cul}$$

where V_0=29.87 MeV, α=0.22 Fm^{-2}, and the Coulomb potential V_{cul} was taken in the customary form with $R_{cul} = 0$ [4]. Here constant $\hbar^2 / m_0 = 41.80159$, where m_0 is the atomic mass unit [14].

The results of the S-phase shift calculation with such potential are shown in fig. 3 with a dotted line. The phase shift reaches the value of 90^0 at energy $E_{p,\,lab.}$ = 467 keV, what is in a fairly good agreement with the experimental value of the resonance at $E_{p,\,lab.}$ = 461 (3) keV [15].

The total cross sections of the elastic scattering obtained by means of the expression (7), are presented with points in fig. 6. They are calculated on the basis of S – phase shifts of the elastic scattering, extracted from the experimental differential cross sections [1]. In fig. 6 the total cross sections, obtained by using the phase shifts calculated with the mentioned above potential, are shown with a continuous line. The open squares represent the total cross sections obtained from the phase shift analysis of work [11].
The obtained potential has the simplest form and does not contain permitted (PS) or forbidden bound states (FS). The ground state of ^{13}N nucleus has spin and parity $J^P = 1/2^-$ and

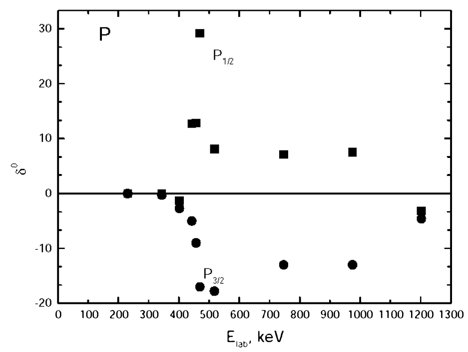


Figure 5. P – phase shift of the $p^{12}C$ elastic scattering at low energies. Points ($P_{3/2}$) and squares ($P_{1/2}$) are the results of the phase shift analysis for P – phase shift with S and P - waves taking into account

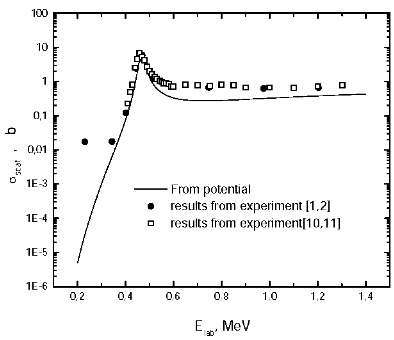


Figure 6. Total cross sections of the $p^{12}C$ elastic scattering

lies 1.94 MeV below the threshold of the $p^{12}C$ channel [15]. So, in case of examination the bound state (BS) of ^{13}N in the framework of the $p^{12}C$ model, it must be present in $P_{1/2}$ – wave of the $p^{12}C$ potential with bond energy 1.94 MeV.

In fig. 6, the plateau in the total cross sections of the elastic scattering is observed within the energy range $E_{p,\,lab.}$ = 200-300 keV, and thus, the same is observed in the S – phase shift of the scattering. At present, it is impossible to understand whether it results from the experimental uncertainties, unexpected errors of the phase shift analysis or the plateau really exists within this range of energies. To clear up this matter it is necessary to take new measurements of angular distributions at energies from 100 to 350 keV with the energy intervals about 50 keV or new measurements of excitation functions at different angles.

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