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ERDOĞAN ŞUHUBİ
(İstanbul Teknik Üniversite Nükleer Enerji Enstitüsü, Turk

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# The most general equations for multigroup neutron diffusion in $P_1$ approximation

By **Erdoğan Suhubi**

(İstanbul Teknik Üniversite Nükleer Enerji Enstitüsü, Turkey)

and **Ahmed Yüksel Özemre**

(Çekmece Nükleer Arastirma ve Eğitim Merkezi; İstanbul, Turkey)

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*Summary.* Starting from the general transport equation and within the limits of $P_1$ approximation, a set of coupled second order partial integro-differential equations is obtained. These equations consider the effect of $m$ groups of retarded neutrons and contain both the first and the second derivatives of the neutron fluxes with respect to the time variable.

*Nomenclature*

| | |
|---|---|
| $\varphi(\vec{r}, E, \vec{\Omega}, t)$ | flux of neutrons of energy $E$ and direction $\vec{\Omega}$ at the point of space coordinates $\vec{r}$ and the time $t$. |
| $\Sigma_{\text{tot}}(\vec{r}, E)$ | macroscopic total cross section of neutrons of energy $E$ at the point of space coordinates $\vec{r}$. |
| $\Sigma_f(\vec{r}, E)$ | macroscopic cross section for fission originated by neutrons of energy $E$ at the point of space coordinates $\vec{r}$. |
| $\Sigma_s(\vec{r}; E', \vec{\Omega}' \to E, \vec{\Omega})$ | macroscopic cross section for scattering of neutrons whose energy and direction change from $E'$ and $\vec{\Omega}'$ to $E$ and $\vec{\Omega}$ as a result of the scattering they suffer at the point of space coordinates $\vec{r}$. |
| $f_0(E)$ | fission spectrum of neutrons. |
| $f_k(E)$ | spectrum of the $k$-th group of retarded neutrons. |
| $\beta(E)$ | fraction of the retarded neutrons which appear at the energy $E$. |
| $\beta_k(E)$ | fraction of the $k$-th group retarded neutrons which appear at the energy $E$. |
| $\nu(E)$ | number of prompt neutrons of energy $E$. |
| $\lambda_k$ | desintegration constant of the $k$-th mother nucleus. |
| $v$ | velocity of neutrons. |
| $Q(\vec{r}, E, \vec{\Omega}, t)$ | source of neutrons of energy $E$ and direction $\vec{\Omega}$ at the point of space coordinates $\vec{r}$ and time $t$. |

The general transport equation which also accounts on the effect of various groups of retarded neutrons may be written as follows [1]:

$$\left.\begin{aligned}
\frac{1}{v}\frac{\partial\varphi(\vec{r},E,\vec{\Omega},t)}{\partial t} &= -\vec{\Omega}\cdot\overrightarrow{\operatorname{grad}}\,\varphi(\vec{r},E,\vec{\Omega},t) - \\
&-\Sigma_{\text{tot}}(\vec{r},E)\,\varphi(\vec{r},E,\vec{\Omega},t) + \\
&+\int_0^\infty\int_{\vec{\Omega}'}\Sigma_s(\vec{r};E',\vec{\Omega}'\to E,\vec{\Omega})\times \\
&\times\varphi(\vec{r},E',\vec{\Omega}',t)\,dE'\,d\vec{\Omega}' + \\
&+\frac{f_0(E)}{4\pi}\int_0^\infty\int_{\vec{\Omega}'}\nu(E')\,[1-\beta(E')]\times \\
&\times\Sigma_f(\vec{r},E')\,\varphi(\vec{r},E',\vec{\Omega}',t)\,dE'\,d\vec{\Omega}' + \\
&+\frac{1}{4\pi}\sum_{k=1}^{m}\lambda_k f_k(E)\int_0^t\exp[-\lambda_k(t-t')]\times \\
&\times\int_0^\infty\int_{\vec{\Omega}'}\beta_k(E')\,\nu(E')\,\Sigma_f(\vec{r},E')\times \\
&\times\varphi(\vec{r},E',\vec{\Omega}',t')\,dE'\,d\vec{\Omega}'\,dt' + Q(\vec{r},E,\vec{\Omega},t).
\end{aligned}\right\}\quad(1)$$

Now, let us divide the energy interval between the thermal energy and the maximum energy of the fission neutrons in $N$ subintervals. We will define the neutron groups as follows: all the neutrons whose energies lie within the energy interval $(E_{i-1}, E_i)$, $E_i < E_{i-1}$, will be called to form the $i$-th group of neutrons; according to this definition the thermal group will then coincide with the $N$-th group.

The flux of neutrons belonging to the group $i$ will be defined as

$$\varphi_i(\vec{r}, \vec{\Omega}, t) = \int_{E_{i-1}}^{E_i} \varphi(\vec{r}, E, \vec{\Omega}, t)\, dE. \quad (2)$$

The value of macroscopic total or fission cross section for the group $i$ may be obtained by averaging with respect to $\varphi(\vec{r}, E, \vec{\Omega}, t)$:

$$\Sigma_i(\vec{r}) = \frac{\int_{E_{i-1}}^{E_i} \Sigma(\vec{r}, E) \cdot \varphi(\vec{r}, E, \vec{\Omega}, t)\, dE}{\int_{E_{i-1}}^{E_i} \varphi(\vec{r}, E, \vec{\Omega}, t)\, dE}. \tag{3}$$

Similarily, the cross section for transfer from the group $j$ to the group $i$ may be defined as:

$$\left.\begin{aligned} &\Sigma_{j\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega}) \\ &= \frac{\int_{E_{j-1}}^{E_j} dE' \varphi(\vec{r}, E', \vec{\Omega}', t) \int_{E_{i-1}}^{E_i} dE\, \Sigma_s(\vec{r}; E', \vec{\Omega}' \to E, \vec{\Omega})}{\int_{E_{j-1}}^{E_j} \varphi(\vec{r}, E', \vec{\Omega}', t)\, dE'}. \end{aligned}\right\} \tag{4}$$

The number $\nu_i$ of secondery neutrons for the group $i$, the fraction $\beta_i$ of all the retarded neutrons and the fraction $\beta_{k,i}$ of the $k$-th group of retarded neutrons which born in the group $i$ are respectively defined by the following expressions:

$$\left.\begin{aligned} \nu_i &= \int_{E_{i-1}}^{E_i} \nu(E)\, dE \\ \beta_i &= \int_{E_{i-1}}^{E_i} \beta(E)\, dE \\ \beta_{k,i} &= \int_{E_{i-1}}^{E_i} \beta_k(E)\, dE. \end{aligned}\right\} \tag{5}$$

The values of the fission and retarded neutrons spectra corresponding to the group $i$ are respectively

$$\left.\begin{aligned} f_{0,i} &= \int_{E_{i-1}}^{E_i} f_0(E)\, dE \\ f_{k,i} &= \int_{E_{i-1}}^{E_i} f_k(E)\, dE. \end{aligned}\right\} \tag{6}$$

When all the terms of the transport equation (1) are integrated between the limits $E_{i-1}$ and $E_i$, with respect to $E$, and because of the formulas (2 to 6) and of the discontinuity of the energy groups which permits the substitution of the summations on the groups instead of the integrals with respect to the variable $E'$, the transport equation may be written as the following set of integro-differential equations:

$$\left.\begin{aligned} \frac{1}{v_i} \frac{\partial \varphi_i(\vec{r}, \vec{\Omega}, t)}{\partial t} &= -\vec{\Omega} \cdot \overrightarrow{\operatorname{grad}}\, \varphi_i(\vec{r}, \vec{\Omega}, t) - \\ &- \Sigma_{\mathrm{tot},i}(\vec{r})\, \varphi_i(\vec{r}, \vec{\Omega}, t) + \\ &+ \sum_{j=1}^{i} \int_{\vec{\Omega}'} \Sigma_{j\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega})\, \varphi_j(\vec{r}, \vec{\Omega}', t)\, d\vec{\Omega}' + \\ &+ \frac{f_{0,i}}{4\pi} \sum_{j=1}^{N} \int_{\vec{\Omega}'} (1-\beta_j)\, \nu_j\, \Sigma_{f,j}\, \varphi_j(\vec{r}, \vec{\Omega}', t)\, d\vec{\Omega}' + \\ &+ \frac{1}{4\pi} \sum_{k=1}^{m} \lambda_k f_{k,i} \sum_{j=1}^{N} \int_0^{\infty} \int_{\vec{\Omega}'} \beta_{k,j}\, \nu_j\, \Sigma_{f,j} \times \\ &\times \varphi_j(\vec{r}, \vec{\Omega}', t') \exp[-\lambda_k (t-t')]\, d\vec{\Omega}'\, dt' + \\ &+ Q_i(\vec{r}, \vec{\Omega}, t) \qquad (i = 1, 2, \ldots, N) \end{aligned}\right\} \tag{7}$$

where $v_i$ is the average velocity of neutrons corresponding to the group $i$.

We now denote the time-dependent scalar neutron flux by

$$\varphi_i(\vec{r}, t) = \int_{\vec{\Omega}} \varphi_i(\vec{r}, \vec{\Omega}, t)\, d\vec{\Omega} \tag{8}$$

and the neutron current by

$$\vec{J}_i(\vec{r}, t) = \int_{\vec{\Omega}} \vec{\Omega} \cdot \varphi_i(\vec{r}, \vec{\Omega}, t)\, d\vec{\Omega}. \tag{9}$$

In the $P_1$ approximation [2] we have

$$\varphi_i(\vec{r}, \vec{\Omega}, t) = \frac{1}{4\pi} \varphi_i(\vec{r}, t) + \frac{3}{4\pi} \vec{\Omega} \cdot \vec{J}_i(\vec{r}, t). \tag{10}$$

After having substituted this expression into eq. (7), let us integrate the equation first over all the directions and then after having multiplied scalarly with $\vec{\Omega}$. Thus we obtain

$$\left.\begin{aligned} \frac{1}{v_i} \frac{\partial \varphi_i(\vec{r}, t)}{\partial t} &= -\operatorname{div} \vec{J}_i(\vec{r}, t) - \\ &- \Sigma_{\mathrm{tot},i}(\vec{r})\, \varphi_i(\vec{r}, t) + \sum_{j=1}^{i} \Sigma_{j\to i}(\vec{r})\, \varphi_j(\vec{r}, t) + \\ &+ f_{0,i} \sum_{j=1}^{N} (1-\beta_j)\, \nu_j\, \Sigma_{f,j}(\vec{r})\, \varphi_j(\vec{r}, t) + \\ &+ \sum_{k=1}^{m} \lambda_k f_{k,i} \sum_{j=1}^{N} \beta_{k,j}\, \nu_j\, \Sigma_{f,j}(\vec{r}) \int_0^t \varphi_j(\vec{r}, t') \times \\ &\times \exp[-\lambda_k (t-t')]\, dt' + Q_i(\vec{r}, t), \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} \frac{1}{v_i} \frac{\partial \vec{J}_i(\vec{r}, t)}{\partial t} &= -\frac{1}{3} \overrightarrow{\operatorname{grad}}\, \varphi_i(\vec{r}, t) - \\ &- \Sigma_{\mathrm{tot},i}(\vec{r})\, \vec{J}_i(\vec{r}, t) + \sum_{j=1}^{i} \mu_{ji}\, \Sigma_{j\to i}(\vec{r})\, \vec{J}_j(\vec{r}, t) \end{aligned}\right\} \tag{12}$$

where

$$\Sigma_{j\to i}(\vec{r}) = \int_{\vec{\Omega}} \Sigma_{j\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega})\, d\vec{\Omega} \tag{13}$$

and

$$\mu_{ji} = \frac{\int_{\vec{\Omega}} \Sigma_{j\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega}) \cos(\vec{\Omega}', \vec{\Omega})\, d\vec{\Omega}}{\int_{\vec{\Omega}} \Sigma_{j\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega})\, d\vec{\Omega}}. \tag{14}$$

When we consider the macroscopic cross section for transfer from the group $j$ to the group $i$, we, immediately, see that $j$ can not be greater than $i$; and for $j = 1$, $\Sigma_{i\to i}$ where

$$\Sigma_{i\to i}(\vec{r}) = \int_{\vec{\Omega}} \Sigma_{i\to i}(\vec{r}, \vec{\Omega}' \to \vec{\Omega})\, d\vec{\Omega},$$

represents only the macroscopic cross section for directional changes in the same group. This is a special feature of the $P_1$ approximation which can not be encountered in the elementary diffusion theory. For example OKRENT's equations [3] do not contain such a term. The presence of this term in the diffusion equations involves an apperent decrease of the macroscopic total cross sections. Indeed, if we put

$$\tilde{\Sigma}_{\mathrm{tot},i}(\vec{r}) = \Sigma_{\mathrm{tot},i}(\vec{r}) - \Sigma_{i\to i}(\vec{r})$$

which is obviously smaller than the macroscopic total cross section $\Sigma_{\mathrm{tot},i}(\vec r)$, the equations (11) and (12) may be written as

$$\left.\begin{aligned} \frac{1}{v_i}\frac{\partial\varphi_i(\vec r,t)}{\partial t} &= -\operatorname{div}\vec J_i(\vec r,t) - \\ &- \tilde\Sigma'_{\mathrm{tot},i}(\vec r)\,\varphi_i(\vec r,t) + \sum_{j=1}^{i-1}\Sigma_{j\to i}(\vec r)\,\varphi_j(\vec r,t) + \\ &+ f_{0,i}\sum_{j=1}^{N}(1-\beta_j)\,\nu_j\,\Sigma_{f,j}(\vec r)\,\varphi_j(\vec r,t) + \\ &+ \sum_{k=1}^{m}\lambda_k f_{k,i}\sum_{j=1}^{N}\beta_{kj}\nu_j\Sigma_{f,j}(\vec r)\int_0^t\varphi_j(\vec r,t')\times \\ &\times\exp[-\lambda_k(t-t')]\,dt' + Q_i(\vec r,t), \end{aligned}\right\}\quad(11')$$

$$\left.\begin{aligned} \frac{1}{v_i}\frac{\partial\vec J_i(\vec r,t)}{\partial t} &= -\frac{1}{3}\,\overrightarrow{\operatorname{grad}}\,\varphi_i(\vec r,t) - \\ &- [\Sigma_{\mathrm{tot},i}(\vec r) - \mu_{ii}\Sigma_{i\to i}(\vec r)]\,\vec J_i(\vec r,t) + \\ &+ \sum_{j=1}^{i-1}\mu_{ji}\Sigma_{j\to i}(\vec r)\,\vec J_j(\vec r,t) \end{aligned}\right\}\quad(12')$$

and here $\tilde\Sigma_{\mathrm{tot},i}(\vec r)$ plays the rôle of the macroscopic total cross section.

If we put

$$D_{ii} = \frac{1}{3(\Sigma_{\mathrm{tot},i} - \mu_{ii}\Sigma_{i\to i})} \qquad (15)$$

the expression (12′) may be reduced into the following form

$$\left.\begin{aligned} \vec J_i(\vec r,t) = -D_{ii}\Big[&\overrightarrow{\operatorname{grad}}\,\varphi_i(\vec r,t) + \frac{3}{v_i}\frac{\partial\vec J_i(\vec r,t)}{\partial t} - \\ &- 3\sum_{j=1}^{i-1}\mu_{ji}\Sigma_{j\to i}\vec J_j(\vec r,t)\Big]. \end{aligned}\right\}\quad(16)$$

For $i=1$, we have:

$$\vec J_1(\vec r,t) = -D_{11}\left[\overrightarrow{\operatorname{grad}}\,\varphi_1(\vec r,t) + \frac{3}{v_1}\frac{\partial\vec J_1(\vec r,t)}{\partial t}\right],$$

and putting

$$D_{12} = 3D_{22}D_{11}\mu_{12}\Sigma_{1\to 2} \qquad (17)$$

we obtain for $i=2$

$$\begin{aligned} \vec J_2(\vec r,t) &= -D_{22}\left[\overrightarrow{\operatorname{grad}}\,\varphi_2(\vec r,t) + \frac{3}{v_2}\frac{\partial\vec J_2(\vec r,t)}{\partial t}\right] - \\ &\quad - D_{21}\left[\overrightarrow{\operatorname{grad}}\,\varphi_1(\vec r,t) + \frac{3}{v_1}\frac{\partial\vec J_1(\vec r,t)}{\partial t}\right], \end{aligned}$$

so we may see by induction that, in general, the following expression holds:

$$\vec J_i(\vec r,t) = -\sum_{j=1}^{i} D_{ij}\left[\overrightarrow{\operatorname{grad}}\,\varphi_j(\vec r,t) + \frac{3}{v_j}\frac{\partial\vec J_j(\vec r,t)}{\partial t}\right]. \qquad (18)$$

In this expression all the $D_{ij}$'s can be determined as in eq. (17). If the energy intervals are choosen large enough with respect to the average loss of energy of neutrons in collisions with nuclei, the off diagonal terms of $D_{ij}$ can be, in general, neglected [4], and eq. (18) reduces into:

$$\vec J_i(\vec r,t) = -D_i\left[\overrightarrow{\operatorname{grad}}\,\varphi_i(\vec r,t) - \frac{3}{v_i}\frac{\partial\vec J_i(\vec r,t)}{\partial t}\right]. \qquad (19)$$

Taking the divergence of both the sides of eq. (18) and after rearrangement, we may write:

$$\left.\begin{aligned} -3\sum_{j=1}^{i}\frac{D_{ij}}{v_j}\operatorname{div}\frac{\partial\vec J_i(\vec r,t)}{\partial t} - \operatorname{div}\vec J_i(\vec r,t) \\ = \sum_{j=1}^{i} D_{ij}\,\nabla^2\varphi_j(\vec r,t) \end{aligned}\right\}\quad(20)$$

where we assumed that all the $D_{ij}$'s were independent of the space coordinates.

Now, if we write eq. (11) for the group $j$ and multiply its derivative with respect to time by $3D_{ij}/v_j$, then add all the terms from $j=1$ up to $j=i$ and again add term by term eq. (11) to the so formed expression, we obtain with the aid of the eq. (20) the following set of integro-differential equations which, indeed, are the most general set of equations for multigroup neutron diffusion in the $P_1$ approximation:

$$\left.\begin{aligned} 3\sum_{j=1}^{i}\frac{D_{ij}}{v_j^2}&\frac{\partial^2\varphi_j(\vec r,t)}{\partial t^2} + \frac{1}{v_i}\frac{\partial\varphi_i(\vec r,t)}{\partial t} + \\ &+ 3\sum_{j=1}^{i}\frac{D_{ij}}{v_j}\Big\{\tilde\Sigma_{\mathrm{tot},j}\frac{\partial\varphi_j(\vec r,t)}{\partial t} - \\ &- \sum_{l=1}^{j-1}\Sigma_{l\to j}\frac{\partial\varphi_l(\vec r,t)}{\partial t} - \\ &- f_{0,j}\sum_{l=1}^{N}(1-\beta_l)\,\nu_l\,\Sigma_{f,l}\frac{\partial\varphi_l(\vec r,t)}{\partial t}\Big\} \\ &= \sum_{j=1}^{i} D_{ij}\,\nabla^2\varphi_j(\vec r,t) - \tilde\Sigma_{\mathrm{tot},i}\,\varphi_i(\vec r,t) + \\ &+ \sum_{j=1}^{i-1}\Sigma_{j\to i}\,\varphi_j(\vec r,t) + \\ &+ f_{0,i}\sum_{j=1}^{N}(1-\beta_j)\,\nu_j\,\Sigma_{f,j}\,\varphi_j(\vec r,t) + \\ &+ \sum_{k=1}^{m}\lambda_k f_{k,i}\sum_{j=1}^{N}\beta_{k,j}\,\nu_j\,\Sigma_{f,j}\times \\ &\times\int_0^t\varphi_j(\vec r,t')\exp[-\lambda_k(t-t')]\,dt' + \\ &+ \sum_{j=1}^{i}\frac{3D_{ij}}{v_j}\sum_{k=1}^{m}\lambda_k f_{k,j}\sum_{l=1}^{N}\beta_{k,l}\,\nu_l\,\Sigma_{f,l}\times \\ &\times\Big[\varphi_l(\vec r,t) - \lambda_k\int_0^t\varphi_l(\vec r,t')\times \\ &\times\exp[-\lambda_k(t-t')]\,dt'\Big] + Q_i(\vec r,t) + \\ &+ \sum_{j=1}^{i}\frac{3D_{ij}}{v_j}\frac{\partial Q_j(\vec r,t)}{\partial t} \qquad (i=1,2,\ldots,N). \end{aligned}\right\}\quad(21)$$

These equations look very complicated. However, some simplifications could be realized. For example: 1) the retarded neutrons are found only in the last few groups; so, for many groups $\beta_{k,i}$ and $\beta_i$ vanish; 2) for some nuclear fuel, such as U-238, the fission cross section is different from zero only for high energy, and 3) for some other fuel, such as U-235, the fission spectrum does not extend as far as to the thermal region.

Another simplification arises when we have $D_{ij}=0$ for $i \neq j$, the circumstance of which has already been mentioned. If this condition is realized, considering (11) and (19), it is plausible to see that we may write:

$$\left.\begin{aligned}
&\frac{1}{v_i}\frac{\partial}{\partial t}\left[\varphi_i(\vec r,t)+\frac{3D_i}{v_i}\frac{\partial\varphi_i(\vec r,t)}{\partial t}\right]\\
&\quad= D_i \nabla^2\varphi_i(\vec r,t)-\\
&\quad-\tilde\Sigma_{\mathrm{tot},i}\left[\varphi_i(\vec r,t)+\frac{3D_i}{v_i}\frac{\partial\varphi_i(\vec r,t)}{\partial t}\right]+\\
&\quad+\sum_{j=1}^{i-1}\Sigma_{j\to i}\left[\varphi_j(\vec r,t)+\frac{3D_i}{v_i}\frac{\partial\varphi_j(\vec r,t)}{\partial t}\right]+\\
&\quad+f_{0,i}\sum_{j=1}^{N}(1-\beta_j)\,\nu_j\,\Sigma_{f,j}\times\\
&\quad\times\left[\varphi_j(\vec r,t)+\frac{3D_i}{v_i}\frac{\partial\varphi_j(\vec r,t)}{\partial t}\right]+\\
&\quad+\sum_{k=1}^{m}\lambda_k f_{k,i}\sum_{j=1}^{N}\beta_{k,j}\,\nu_j\,\Sigma_{f,j}\left[\frac{3D_i}{v_i}\varphi_j(\vec r,t)+\right.\\
&\quad+\left(1-\frac{3D_i}{v_i}\lambda_k\right)\int_0^t\varphi_j(\vec r,t')\times\\
&\quad\left.\times\exp[-\lambda_k(t-t')]\,dt'\right]+Q_i(\vec r,t)+\\
&\quad+\frac{3D_i}{v_i}\frac{\partial Q_i(\vec r,t)}{\partial t}\qquad(i=1,2,\ldots,N).
\end{aligned}\right\}\quad(22)$$

If we consider the stationary distribution of neutron fluxes and if we assume that the retarded neutrons are negligible, we obtain

$$\left.\begin{aligned}
&D_i\nabla^2\varphi_i(\vec r)-\tilde\Sigma_{\mathrm{tot},i}\,\varphi_i(\vec r)+\sum_{j=1}^{i-1}\Sigma_{j\to i}\,\varphi_j(\vec r)+\\
&\qquad+f_{0,i}\sum_{k=1}^{N}\nu_k\,\Sigma_{f,k}\,\varphi_k(\vec r)+Q_i(\vec r)=0\\
&\qquad(i=1,2,\ldots,N)
\end{aligned}\right\}\quad(23)$$

which differ from the OKRENT's equations [3] only in the interpretation of the second term and where all the $\Sigma_{j\to i}$'s are supposed to represent the elastic and the inelastic moderation together.

We leave the study of the system (22) in a subsequent paper.

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