THE SOLUTION OF THIRD FORM OF THE TRANSPORT EQUATION USING THE SINGULAR EIGENFUNCTION METHOD: CONSTANT SOURCE PROBLEM

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ABSTRACT

The third form of the transport equation has been solved using C_N method. In this method the infinite medium Green's function which has been obtained from Fourier transform technique was used. The singular eigenfunction method is based on the use of the infinite medium Green's function. The method which we want to use in this calculation is based on the infinite medium Green's function. This function can be written in terms of singular eigenfunctions of the Case's method. This method leads to obtain analytical expression of the problem easily. The numerical results are in good agreement with F_N method.

INTRODUCTION

The neutron transport equation is employed to solve some physical problems as albedo, constant source problem, criticality, extrapolation lengths for Milne problem for isotropic and anisotropic scattering cases. There are three forms of neutron transport equation. The first form which is also known as Boltzmann equation is solved by using the methods of Case, F_N, P_L. The second form is solved by using variational method and the third form of the neutron transport equation is solved by using C_N method.

The C_N method is based on Placzek lemma in which the finite medium is converted into the infinite medium. An integral equation is provided for the angular flux at the boundaries of the various media which allows the fluxes to be calculated at any point [1,2]. For plane geometry C_N equations can be written as

$$\nu^-(\mu) = \int_0^\infty dx' \int_{-1}^1 S(x',\mu_0) G(0,\mu,x',\mu_0) d\mu_0 + \int_0^1 G(\mu,\mu_0)\nu^+(\mu_0)\mu_0 d\mu_0 + \int_{-1}^0 G(\mu,\mu_0)\nu^-(\mu_0)\mu_0 d\mu_0 \qquad \mu<0 \tag{1}$$

$$0 = \int_0^\infty dx' \int_{-1}^1 S(x',\mu_0) G(0,\mu,x',\mu_0) d\mu_0 + \int_0^1 G(\mu,\mu_0)\nu^+(\mu_0)\mu_0 d\mu_0 + \int_{-1}^0 G(\mu,\mu_0)\nu^-(\mu_0)\mu_0 d\mu_0 \qquad \mu>0 \tag{2}$$

These are the Fredholm equations of the second and first type respectively. Where G is the infinite medium Green's function and $G(x,\mu,\mu_0)$ is angular flux at x in the direction μ produced by a plane unit source at the origin, emitting in the direction μ_0. It is obtained from Fourier transformation technique as

$$G(x,\mu,\mu_0) = \frac{c}{4\pi}\int_{-\infty}^{\infty} \frac{e^{-ikx}dk}{(1-ik\mu)(1-ik\mu_0)\left(1-\frac{c\tan^{-1}k}{k}\right)} \tag{3}$$

In our method, the infinite medium Green's function which is obtained from the solution of the Boltzmann equation is used to solve C_N equations [3]. The Green's function $G(x_0 \to x; \mu_0 \to \mu)$ represents the angular flux at x,μ due to a unit plane source at x_0,μ_0. For isotropic scattering, [4]

$$\mu\frac{\partial G(x_0\to x;\mu_0\to\mu)}{\partial x}+G(x_0\to x;\mu_0\to\mu)=\frac{c}{2}\int_{-1}^{1}G(x_0\to x;\mu_0\to\mu')d\mu' + \delta(x-x_0)\delta(\mu-\mu_0) \tag{4}$$

$$G^{\mp}(x_0\to x;\mu_0\to\mu)=\frac{\phi(\pm\nu_0,\mu)\phi(\pm\nu_0,\mu_0)}{N(\nu_0)}e^{-\frac{|x-x_0|}{\nu_0}}+\int_0^1\frac{\phi(\pm\nu,\mu)\phi(\pm\nu,\mu_0)}{N(\nu)}e^{-\frac{|x-x_0|}{\nu}}d\nu \tag{5}$$

where the upper sign is for $x < x_0$, the lower is for $x > x_0$ and

$$\phi(\pm\nu_0,\mu)=\pm\frac{c\nu_0}{2}\frac{1}{\pm\nu_0-\mu}, \tag{6}$$

$$\phi(\nu,\mu)=\frac{c\nu}{2}P\frac{1}{\nu-\mu}+\lambda(\nu)\delta(\nu-\mu), \qquad \lambda(\nu)=1-c\nu\,\mathbf{tanh}^{-1}\nu \tag{7}$$

are known as the discrete and continuous singular eigenfunctions corresponding to the discrete and continuum eigenvalues ν_0 and ν respectively. The following equations

$$N(\pm\nu_0)=\pm\frac{c\nu_0^3}{2}\left[\frac{c}{\nu_0^2-1}-\frac{1}{\nu_0^2}\right] \tag{8}$$

$$N(\nu)=\nu\left[\left(1-c\nu\tanh^{-1}\nu\right)^2+\frac{c^2\pi^2\nu^2}{4}\right] \tag{9}$$

correspond to the normalization constants of the discrete and continuum modes.

CONSTANT SOURCE PROBLEM

We consider the half space constant source problem in Eq.(1), for $c<1$, and the constant source is defined by [5,6]

$$S(x,\mu_0)=a \tag{10}$$

the outgoing angular flux is defined by

$$\nu^-(\mu)=\sum_{l=0}^{N}\nu_l^-\mu^l \tag{11}$$

the incident angular flux is

$$\nu^+(\mu)=0. \tag{12}$$

$a=1$ yields the usual constant–source problem, the half–space albedo problem is obtained by taking $a=0$. Writing Eq.(1) as (where G replaced by $G^{\pm}$)

$$\nu^-(\mu)=\int_0^{\infty}dx'\int_{-1}^{1}S(x',\mu_0)G^-(0,\mu,x',\mu_0)d\mu_0+\int_0^1G^+(\mu,\mu_0)\nu^+(\mu_0)\mu_0d\mu_0 + \int_{-1}^{0}G^+(\mu,\mu_0)\nu^-(\mu_0)\mu_0d\mu_0 \qquad \mu<0 \tag{13}$$

using the infinite medium Green's function given in Eq.(5) and multiplying this equation by μ^{m+1} and integrating over $\mu\in(-1,0)$, we obtain

$$\sum_{l=0}^{N}\nu_l(-1)^l\left[\frac{1}{m+l+1}+\frac{c^2\nu_0^2}{4}\frac{A_m(\nu_0)A_l(\nu_0)}{N(\nu_0)}+\frac{c^2}{4}\int_0^1\nu^2\frac{A_m(\nu)A_l(\nu)}{N(\nu)}d\nu\right]= a\frac{c\nu_0^2}{2}\frac{B_m(\nu_0)}{N(\nu_0)}+\frac{ac}{2}\int_0^1\nu^2\frac{B_m(\nu)}{N(\nu)}d\nu \tag{14}$$

where

$$A_l(z)=\frac{2}{cz}\int_0^1\mu^{l+1}\phi(-z,\mu)d\mu, \tag{15}$$

$$B_l(z)= \frac{2}{cz}\int_0^1 m^{l+1} f(z,m)\; dm \,, \tag{16}$$

$$A_0(\zeta)=1-\zeta \ln\left(1+\frac{1}{\zeta}\right), \tag{17}$$

$$B_0(\zeta)=\frac{2}{c}-1-\zeta \ln\left(1+\frac{1}{\zeta}\right), \tag{18}$$

$$A_l(z)= \frac{1}{1+1}- z A_{l-1}(z) \tag{19}$$

$$B_l(z)= z B_{l-1}(z)- \frac{1}{l+1} \tag{20}$$

with $\zeta = v_0$ or v. We give our results for the parameter of

$$b = - 2\int_{-1}^{0} m n^{-}(m) dm= \sum_{l=0}^{N} \frac{(-1)^l v_l^{-}}{l+2}. \tag{21}$$

We can also use Eq.(2) to calculate b multiplying by m^{m+1} and integrating over $m \in (0,1)$.

Table 1: b for the selected c values for $a = 1$

N	c=0.1	c=0.3	c=0.5	c=0.7	c=0.9
0	1.08687	1.32221	170664	2.47642	5.20568
1	1.08687	1.32224	1.70690	2.47807	5.21946
2	1.08687	1.32225	1.70691	2.47814	5.21974
3	1.08687	1.32225	1.70692	2.47815	5.21975
4	1.08687	1.32225	1.70692	2.47815	5.21976
5	1.08687	1.32225	1.70692	2.47815	5.21976
Siewert	1.087	1.322	1.707	2.478	5.220

CONCLUSIONS

The purpose of the present work is to solve the half-space albedo and the half-space constant source problem for isotropic scattering kernel by using C_N equations. Some of the results are compared with those already existing in literature. In general one can see that some exact results are obtained in the lowest order approximations. This procedure gives us an analytical expression for solving numerically easily.

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