Inflation and reheating predictions of minimally coupled β-exponential potential with an R^2 term in the Palatini formulation

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Abstract

In this work, we focus on the inflationary predictions of β-exponential potential models, where the field delineating the size of extra-dimension is represented as the inflaton. We include an R^2 term in the Palatini gravity since it provides a well-motivated starting point for the analysis of physics at very high energies. Furthermore, the inflaton oscillates around the minimum of the inflation potential until the universe is reheated at the end of the inflationary epoch. This occurs during the reheating phase, at which inflaton decays into the Standard Model particles, which filled the universe. Regardingly, we extend our examination by taking into consideration the reheating effects on inflationary observables. Assuming the standard thermal history after inflation, we present the inflationary predictions, $n_s, r, \mathrm{d}n_s/\mathrm{d}\ln k$ of β-exponential potential with minimal coupling in Palatini R^2 gravity by considering the reheating cases. We show that this specific kind of model allows one to study a wide range of predictions to have a better analysis for the inflationary era by adjusting the model parameters, α, β, λ. In addition, different kinds of constraints from a variety of observations, such as BICEP/Keck, Planck 2018, the Baryon Acoustic Oscillations (BAO), as well as future possible detectable constraints by CMB-S4 are taken into consideration in this study. We find that our results are in good agreement with the recent data and sensitivity forecast for the future CMB-S4.

Keywords: β-exponential inflation, modified gravity, reheating, physics of the early universe, CMBR

1. Introduction

In the field of modern cosmology (Weinberg, 2008), there are a plethora of investigations for many concepts that assist us in gaining a deeper understanding of the universe. Amongst these nomenclatures comes "inflation"; a period of exponential cosmic growth that occurred right after the big bang singularity. Lately, there has been significant attention from the scientific community towards characterizing some properties of the origin of the universe, which can be done through a great understanding of the inflation era, and this is one of the reasons that makes advancing our studies in the concept of inflation so important. Check out (Linde, 2008) for more details about the concept itself. The idea of inflation is very essential to explain a legion of what were believed to be issues for cosmologists, e.g., the structure problem, smoothness problem, flatness problem, and other issues. For more details on how the cosmic concept of inflation participated in solving such issues, see the following studies (Linde, 1983; Starobinsky, 1979). Hence, inflation is considered by the majority to be a milestone for modern cosmology, especially for its merit that it can be extended to explain a wide range of other concepts; see (Mukhanov and Chibisov, 1981) for more detailed cases. Not only does it do that, but the step that took it to a better position of acceptance is that it is supported, regardless of the ongoing debate on the related frameworks, by the observations in the Cosmic Microwave Background (CMB) (Achúcarro et al., 2022).

Since inflation is explained to be growing quasi-exponentially in the early era of the universe, one can relate to the fact that this concept is modeled through the exponential models. In the literature, a variety of different models with different special properties, potentials, and inflation fields are discussed in a very detailed way; see (Martin et al., 2014; Santos da Costa et al., 2021; Benetti et al., 2019; Costa et al., 2018; Benetti and Ramos, 2017). In this work, we are taking into consideration the R^2 term in the Einstein-Hilbert action, which was first introduced by Alexei Starobinsky (Starobinsky, 1980), and it gained acceptance for the fact that this term makes the model naturally predict a period of inflation without needing to introduce a separate inflation field. The R^2 term drives the inflationary expansion, making it one of the earliest and most successful inflationary models.

In Section 2, we begin by introducing the Einstein-Hilbert action, which includes the R^2 term mentioned before this paragraph. Amongst the variations and principles, please see the literature for more deep details (Misner et al., 1973; Wald, 1984), that one can apply to the Einstein-Hilbert action to derive Einstein's equations, we are going to work in this paper with the Palatini formalism [even though it was Einstein who introduced it (Ferraris et al., 1982)]. Palatini formalism is defined as an independent variation with respect to the metric, an independent connection, and reduced standard deviation. It is note-

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worthy to mention that theories based on this formalism satisfy the metric postulates (Will, 2018). One of the merits of the Palatini formalism is that it has been shown to provide interesting phenomenological implications, such as different predictions for the scalar spectral index n_s and tensor-to-scalar ratio r. These features can potentially offer better alignment with observational data from the Cosmic Microwave Background (CMB) and large-scale structure surveys, see the refs. (Gialamas et al., 2023; Borunda et al., 2008; Jana et al., 2023). Thus, by considering this type of formalism/variation principle, our work can leverage the mentioned advantages and, in return, give us a robust and comprehensive analysis of the inflationary dynamics in the context in which our work is set.

In the literature, β-exponential inflation has already been debated in many studies so far. One significant study is the ref. (Alcaniz and Carvalho, 2007), which shows the inflationary predictions of β-exponential potential in a minimally coupled case. They depict the trajectories for different values of β parameters in the $n_s - r$ and the $\mathrm{d}n_s/\mathrm{d}\ln k - n_s$ planes for some chosen values of β and compare them with the bounds available in the cosmological data. In addition, they show the spectral index n_s as a function of β for selected values of the number of e-folds N_*: $47, 54$, and 61. Specifically, they demonstrate that this model predicts acceptable values of the scalar spectral index n_s and of the tensor-to-scalar ratio r for the number of e-folds N_* lying in the interval around $55-60$ and energy scales in the order of the Planck scale, and it admits a wider range of predictions than what conventional exponential scenarios do. They also present that the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$ is possible for both negative and positive values of this potential.

In addition to ref. (Alcaniz and Carvalho, 2007), there are two important studies in the literature that examine the β-exponential inflation; see the following refs. (Santos et al., 2018; dos Santos et al., 2022). Ref. (Santos et al., 2018) shows the $n_s - r$ plane for different values of β, as well as some selected values of λ, considering two different number of e-folds N_*: 50 and 60, and comparing the predictions of this model with the Planck data. Additionally, ref. (dos Santos et al., 2022) studies the non-minimally coupled β-exponential inflation with the CMB data constraints. They present the cosmological consequences of the non-minimally coupled β-exponential inflation with details in the metric formulation. [Recent studies that also include the β-exponential potential model are as follows: (Capistrano et al., 2024; Santos et al., 2023)]. This model of the inflationary potential is very pivotal to take into account because the model can appear in the framework of brane cosmology, where the inflaton, check (Martin et al., 2014) for the models that have been examined so far based on inflaton, is regarded as the field representing the size of extra dimensions. It is suitable to mention here that this type of potential model is derived using the braneworld scenario framework. For more details about the concept of inflation in brane cosmology, please see the following refs. (Santos et al., 2018; Dvali and Tye, 1999). Let us give a few more important studies, regarding: i) Examining the Inflating Branes Concept, see ref. (Mersini-Houghton, 2001), ii) Phenomenological potentials in the 3-brane scenario, see the refs. (Randall and Sundrum, 1999), (Goldberger and Wise, 1999)]. Lastly, the warm inflation scenario is investigated by the class of β-exponential potentials with details in ref. (Santos et al., 2023).

The couplings between the inflaton and the Standard Model (SM) particles are essential in indicating the dynamics of the reheating phase; a phase that transitions the universe from inflationary expansion to a hot, radiation-dominated era; see the following refs. (Abbott et al., 1982; Albrecht et al., 1982; Kofman et al., 1997; Chung et al., 1999; Bassett et al., 2006; Dai et al., 2014; Cook et al., 2015; Munoz and Kamionkowski, 2015; Hanin et al., 2023) for more details about the notion itself. In addition, these couplings result in the production of SM particles leading to the impact on the thermalization process and the subsequent evolution of the universe. The inflaton couples to other fields throughout reheating phase, converting the leftover energy into new particles that make up the radiation energy density (Kofman et al., 1994; Dolgov and Kirilova, 1990; Traschen and Brandenberger, 1990; Saha et al., 2020). Across this manuscript, we thoroughly analyze the reheating effects on the inflationary predictions within the context of our model in order to provide a comprehensive understanding of the reheating dynamics by calculating the inflationary observables for different reheat temperatures. [For details about the reheating dynamics, see the following refs. (Kofman et al., 1994; Amin et al., 2014; Allahverdi et al., 2010)]. Furthermore, in this work, we depict that the inflationary predictions with reheating impacts can have a consistency of our model with current observational data. Check (Drewes, 2016; Aoki et al., 2022; Repond and Rubio, 2016) for more details about the interactions and the exchanged impacts between different models and the inflaton ϕ. Additionally you can check these literature's (Bassett et al., 2006; Kallosh et al., 2000; Podolsky et al., 2006) for further analysis related to the reheating process and the subsequent cosmological observables.

In this manuscript, we study the inflationary predictions for the potential model that generalizes the well-known power law inflation through a general exponential function (Alcaniz and Carvalho, 2007; Martin et al., 2014), which is the β-exponential potential. The framework of braneworld scenarios can be used to generate this potential, which can be accurately compared with the observational data (Alcaniz and Carvalho, 2007; Santos et al., 2018; dos Santos et al., 2022). In this work, we specifically study this potential in the minimally coupled case with an R^2 term in Palatini formalism. We analyze the inflationary observables of this potential and compare them with the current data from Planck and BICEP/Keck (Ade et al., 2021), as well as the future CMB-S4 sensitivity forecast (Abazajian et al., 2019). On the other hand, it is good to mention here, Ref. (Antoniadis et al., 2019), which recently indicates that the underlying theory of gravity is a Palatini rather than being a metric when the gravity sector is extended by an αR^2 term (where α is a dimensionless parameter). This is very crucial because of the theory that is considered non-renormalizable at medium scales due to a substantial non-minimal coupling to gravity (see the ref. (Rubio, 2019)). Additionally, it can be argued that the quantum corrections in curved spacetime generate both a non-minimal coupling of the type $\xi\phi^2 R$ and a Starobinsky-like αR^2 term (see, for example, (Salvio and Mazumdar, 2015)). Therefore, adding an αR^2 term provides a well-motivated starting point for the physics analysis at very high energies (Tenkanen, 2019). In literature, the studies that discuss the Palatini R^2 inflation can be listed as follows: Refs. (Karam et al., 2019; Antoniadis et al., 2018, 2020; Tenkanen, 2019; Karam et al., 2021; Dimopoulos et al., 2022).

The paper is mapped as follows. In Section 2, we introduce the framework we are going to work on, such as introducing the Einstein-Hilbert action that includes the R^2 term in the Palatini formalism for different frames. Moreover, we introduce the $\beta-$exponential model, and its mathematical properties in order to provide a better understanding for our analytical and numerical analysis later on. We also introduce the Einstein frame potential form in the section alongside an illustrated study for a variety of choices of β to get a better image. The slow-roll parameters are provided in both the canonical scalar field σ and in the terms of the scalar field ϕ. The number of e-folds N_* is also given in two different forms for both analytical and numerical calculations, and in order to get better results for the latter one the reheat temperature concept T_{reh} is also introduced, for two different scenarios. In Section 3, we show and discuss our analytical and numerical results. Finally, we conclude the paper in Sect. 4. We adopt $M_{\rm P}$ to unity for our calculations that we depict with details in Section 3.

2. Palatini β-exponential inflation with an R^2 term

In this section, we begin by introducing the action that is considered in this work (Antoniadis et al., 2018, 2019; Tenkanen, 2019)

$$S_J = \int d^4x\sqrt{-g}\left(\frac{1}{2}M_{\rm P}^2 R + \frac{\alpha}{4}R^2 - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\right), \quad (1)$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, J indicates that the action is given in the Jordan frame. R is the Ricci scalar, which is defined by $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$, where $R_{\mu\nu}$ is the Ricci tensor calculated using the Christoffel symbols $\Gamma^\lambda_{\mu\nu}$. Also, ϕ is the scalar field, called the inflaton, and $V(\phi)$ is the potential given in the Jordan frame. M_P is the reduced Planck mass, and α is the dimensionless parameter. The Jordan frame action given in Eq. (1) can be written in terms of the auxiliary scalar field χ dynamically, as follows (Sotiriou and Faraoni, 2010; Antoniadis et al., 2018, 2019; Tenkanen, 2019)

$$\begin{aligned} S_J &= \int d^4x\sqrt{-g}\Big(\frac{1}{2}M_{\rm P}^2\left(1+\alpha\chi^2\right)R - \frac{\alpha}{4}\chi^4 \\ &- \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\Big). \end{aligned} \quad (2)$$

Performing a Weyl transformation of the metric with

$$\tilde{g}_{\mu\nu} \to \Omega g_{\mu\nu}, \qquad \Omega \equiv 1 + \frac{\alpha\chi^2}{M_{\rm P}^2}, \quad (3)$$

one can write the action within the Einstein frame, resulting in (Antoniadis et al., 2019):

$$S_E = \int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{2}M_{\rm P}^2\tilde{R} - \frac{1}{2\Omega}\nabla^\mu\phi\nabla_\mu\phi - \left(\frac{M_{\rm P}}{\Omega}\right)^2 V(\phi,\chi)\right), \quad (4)$$

here $V(\phi,\chi) = V(\phi) + \frac{\alpha}{4}\chi^4$. Variation of this action given in Eq. (4) with respect to the χ is obtained for the constraint equation (Antoniadis et al., 2018, 2019; Enckell et al., 2019)

$$\frac{\delta S_E}{\delta\chi} = 0 \;\to\; \frac{\chi^2}{M_{\rm P}^2} = \frac{4V(\phi) + \nabla^\mu\phi\nabla_\mu\phi}{M_{\rm P}^4 - \alpha\nabla^\mu\phi\nabla_\mu\phi}. \quad (5)$$

Substituting Eq. (5) into Eq. (4), one can obtain the form (Antoniadis et al., 2018, 2019; Tenkanen, 2019)

$$S_E \simeq \int d^4x\sqrt{-\tilde{g}}\left(\frac{1}{2}M_{\rm P}^2\tilde{R} - \frac{1}{2}\frac{\nabla^\mu\phi\nabla_\mu\phi}{\left(1+\frac{4\alpha}{M_{\rm P}^4}V(\phi)\right)} - \frac{V(\phi)}{\left(1+\frac{4\alpha}{M_{\rm P}^4}V(\phi)\right)}\right). \quad (6)$$

Higher than quadratic powers of $\nabla\phi$ are not anticipated to be relevant, since our goal is to examine the features of this action within the context of slow-roll inflation (Antoniadis et al., 2019). Therefore, the terms with higher than quadratic powers of $\nabla\phi$ are ignored in this work.

In addition, with the suitable field redefinition, we can find $\phi = \phi(\zeta)$, by using the following definition

$$d\zeta = \frac{d\phi}{\sqrt{1+\frac{4\alpha}{M_{\rm P}^4}V(\phi)}} = \frac{d\phi}{\sqrt{Z(\phi)}}, \quad (7)$$

where ζ is the canonical scalar field, $Z(\phi)$ is known as the field space metric. With this definition in Eq. (7), the kinetic term can be expressed within a canonical form. This equation can be solved with respect to the structure of the specific potential models. In addition, we can indicate the Einstein frame potential from the action described in Eq. (6) as follows

$$V_E(\phi) = \frac{V(\phi)}{\left(1+\frac{4\alpha}{M_{\rm P}^4}V(\phi)\right)}. \quad (8)$$

In this work, we focus on the inflationary predictions of the β-exponential potential, which we describe in the next section. We display the inflationary parameters for this potential, the spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index $dn_s/d\ln k$, by assuming the standard thermal history after inflation, and for this potential, we show compatible regions for the spectral index n_s and the tensor-to-scalar ratio r within the recent Planck + BICEP/Keck data and the future CMB-S4 sensitivity forecast. We show the cosmological consequences of β-exponential inflation with an R^2 term in Palatini formalism, presenting the results of inflationary predictions of this potential.

2.1. β-exponential inflation

In this work, we study the β-exponential potential model, which was first introduced and studied in ref. (Alcaniz and

Carvalho, 2007) as a generalization of the power law inflation (Abbott and Wise, 1984; Ratra and Peebles, 1988; Ferreira and Joyce, 1998; Martin et al., 2014) phenomenology.

We start constructing our model with the usual exponential function (Martin et al., 2014) which is defined by

$$V(\phi) = M^4 \exp\left(-\lambda\phi/M_{\rm P}\right) . \tag{9}$$

In this work, we discuss a possible generalization for the inflation potential, which is given in Eq. (9), with the following form

$$V(\phi) = M^4 \exp_{1-\beta}\left(-\lambda\phi/M_{\rm P}\right) , \tag{10}$$

where the generalized exponential function $\exp_{1-\beta}$ is defined by (Lima et al., 2001; Alcaniz and Carvalho, 2007; Abramowitz and Stegun, 1965)

$$\exp_{1-\beta}(f) = \left[1+\beta f\right]^{1/\beta} , \tag{11}$$

$$\text{for} \begin{cases} 1+\beta f > 0 \\ \exp_{1-\beta}(f) = 0, \text{ otherwise.} \end{cases}$$

For $f > 0$ and $g > 0$, this function satisfies the following identities (as it is already shown and discussed with details in (Alcaniz and Carvalho, 2007; Martin et al., 2014)):

$$\exp_{1-\beta}\left[\ln_{1-\beta}(f)\right] = f ,$$

and

$$\ln_{1-\beta}(f) + \ln_{1-\beta}(g) = \ln_{1-\beta}(fg) - \beta\left[\ln_{1-\beta}(f)\ln_{1-\beta}(g)\right],$$

where $\ln_{1-\beta}(f) = (f^\beta - 1)/\beta$ is the generalized logarithmic function. The β-exponential potential model was discussed with details in ref. (Alcaniz and Carvalho, 2007), later being prompted to the braneworld framework in (Santos et al., 2018). As we already mentioned above, this model is the generalization of the exponential potential, which can be adjusted with the parameter β, for when $\beta \to 0$, it gives the general exponential function. A wide range of cosmological solutions for β values are presented thanks to such β-exponential potentials (Martin et al., 2014). The β-exponential potential can bring about the breakdown of the slow-roll regime with the end of inflation. In particular, the predictions of this potential model can take the tiny values of the tensor-to-scalar ratio, r (Alcaniz and Carvalho, 2007; Tenkanen, 2019).

The Jordan frame potential, $V(\phi)$, for the β-exponential inflation can be written in the following form

$$V(\phi) = V_0\left(1 - \lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}, \tag{12}$$

where the deviation from the pure exponential function is controlled by constant β, while λ is a dimensionless constant. In the framework of brane cosmology, where the radion (a field characterizing the size of the extra-dimension) is elucidated as the inflaton, the β-exponential potential in Eq. (12) can arise, for more details see ref. (Santos et al., 2018). The important point to emphasize is that, the brane inflation scalar potential can be easily obtained from the effective potential which is described by the four-dimensional action as follows (Santos et al., 2018)

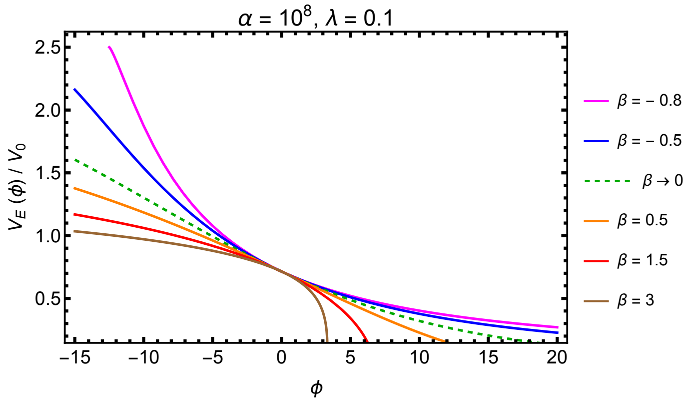

Figure 1: The Einstein frame β-exponential potential with minimal coupling in Palatini R^2 gravity as a function of ϕ. The colors show different values of the β parameter. We fixed $\lambda = 0.1$, $\alpha = 10^8$, $V_0 = 10^{-9}$, as well as taken $M_{\rm P}$ unity.

$$S_4 = \int d^4x\sqrt{-g_4}\left(\frac{1}{2}\sigma\dot{L}^2 - V_{\rm eff}(L)\right) , \tag{13}$$

where σ is the brane tension, L is the position of the brane with respect to $r = 0$, here r is the fifth coordinate of the 5D bulk. Also, the effective potential is defined as $V_{\rm eff}(L) = V_0(1+c_1L)^{\frac{1}{\lambda c_1}} + \frac{1}{2}\sigma$. In addition, with some changes in the definitions of the effective potential, $V_{\rm eff}(L)$, the equivalent form of the β-exponential potential, which is presented in Eq. (12) can be found, see ref. (Santos et al., 2018). According to the studies refs. (Santos et al., 2018; dos Santos et al., 2022), it can be inferred that the brane tension σ is related to the ratio β/λ. Both β and λ are constrained by this connection; that is, β must be greater than λ, with $\beta \geq 1/2$. Thus, considering the β-exponential potential models is highly motivated for the context of braneworld scenarios and brane dynamics (dos Santos et al., 2022).

In this work, with the form of Eq. (8), we study the Einstein frame minimally coupled potential for the β-exponential inflation with an R^2 term in the Palatini formalism. By using Eq. (12), we can define the potential model in the Einstein frame as follows

$$V_E(\phi) = \frac{V_0\left(1-\lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}}{\left(1+4\alpha\frac{V_0\left(1-\lambda\beta\frac{\phi}{M_{\rm P}}\right)^{1/\beta}}{M_{\rm P}^4}\right)}. \tag{14}$$

In Figure 1, we illustrate how the Einstein frame β-exponential potential, which is given in Eq. (14) changes according to the values of the β parameter, which we select and how this parameter controls the potential model, as well as deviation from the usual exponential function.

2.2. *Inflationary observables*

When the Einstein frame potential is being obtained in terms of the canonical scalar field ζ, inflationary predictions can be calculated by using the slow-roll parameters (Lyth and Liddle, 2009)

$$\epsilon = \frac{M_{\rm P}^2}{2}\left(\frac{V_\zeta}{V}\right)^2, \quad \eta = M_{\rm P}^2\frac{V_{\zeta\zeta}}{V}, \quad \kappa^2 = M_{\rm P}^4\frac{V_\zeta V_{\zeta\zeta\zeta}}{V^2}, \tag{15}$$

where ζ's in the subscript denote derivatives according to the scalar field ζ. Inflationary observables, for instance, the spectral index n_s, the tensor-to-scalar ratio r, and the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$ are represented in the following forms for the slow-roll approximation:

$$n_s = 1 - 6\epsilon + 2\eta, \quad r = 16\epsilon,$$
$$\frac{\mathrm{d}n_s}{\mathrm{d}\ln k} = 16\epsilon\eta - 24\epsilon^2 - 2\kappa^2. \tag{16}$$

Recently, accurate constraints on the inflationary predictions were given by BICEP/Keck (Ade et al., 2021), especially for the tensor-to-scalar ratio r, which tightens to $r < 0.035$ at 95% CL. This strong constraint gives an explanation for the amplitude of the primordial gravitational waves as well as the inflationary scale. Moreover, recent BICEP/Keck results also constrain the spectral index n_s to the range $[0.957, 0.976]$ at 2σ of confidence level. These constraints are indicated for the pivot scale $k_* = 0.002$ Mpc^{-1}. Another pivotal constraint arises from the Planck 2018 measurements along with the results from the baryon acoustic oscillations (BAO). They provide the constraint on $\mathrm{d}n_s/\mathrm{d}\ln k = -0.0041 \pm 0.0067$ to base ΛCDM in 68%, TT,TE,EE +lowE+lensing+BAO (Aghanim et al., 2020). It should be noted that the most recent constraints on the $\mathrm{d}n_s/\mathrm{d}\ln k$ are not very robust to test the existing inflationary models accurately. Some improvements are expected from the observations of the 21-cm line in the future (Kohri et al., 2013; Basse et al., 2015; Muñoz et al., 2017).

In addition, for the case of the tensor-to-scalar ratio $r > 0.003$, at larger than 5σ of confidence level, primordial gravitational waves can be detectable in the future by CMB-S4 (Abazajian et al., 2019). The highest limit of the tensor-to-scalar ratio $r < 0.001$ at 95% CL may be reached through future observations done by CMB-S4, even in the absence of a detection, this limit would still provide important new insights for the inflation (Abazajian et al., 2019). It is expected that the CMB-S4 outcomes may supply strong constraints on the inflationary models in the future, and the results from future CMB-S4 might rule out most of the existing inflationary models with much more robust constraints.

The number of e-folds N_* in the slow-roll approximation is given by,

$$N_* = \frac{1}{M_{\rm P}^2}\int_{\zeta_e}^{\zeta_*}\frac{V\mathrm{d}\zeta}{V_\zeta}, \tag{17}$$

where the subscript "$_*$" denotes quantities when the pivot scale exits the horizon, and ζ_e is the value of the inflaton at which the inflation ends, we can compute ζ_e via $\epsilon(\zeta_e) = 1$.

The amplitude of the curvature perturbation can be calculated using the following relation,

$$\Delta_\mathcal{R} = \frac{1}{2\sqrt{3}\pi M_{\rm P}^3}\frac{V^{3/2}}{|V_\zeta|}, \tag{18}$$

the best fit value for the pivot scale $k_* = 0.002$ Mpc^{-1} is $\Delta_\mathcal{R}^2 \approx 2.1\times10^{-9}$ (Aghanim et al., 2020) from the Planck measurement.

On the other hand, it may not always be easy or possible to express an inflation potential defined as $V_J(\phi)$ in the Jordan frame as $V_E(\zeta)$ in the Einstein frame. In this case, the numerical calculations should be made in terms of the original scalar field ϕ instead of the canonical scalar field ζ. We use such equations to perform numerical calculations in terms of the scalar field ϕ when necessary. Furthermore, for the numerical calculations, one needs to have the slow-roll parameters in terms of the field ϕ to be able to calculate the inflationary predictions of the potential model in terms of the general values of free parameters. Thus, the slow-roll parameters should be acquired in terms of the scalar field ϕ, these parameters can be written as follows (Linde et al., 2011)

$$\epsilon = Z\epsilon_\phi, \quad \eta = Z\eta_\phi + \mathrm{sgn}(V')Z'\sqrt{\frac{\epsilon_\phi}{2}},$$
$$\kappa^2 = Z\left(Z\kappa_\phi^2 + 3\mathrm{sgn}(V')Z'\eta_\phi\sqrt{\frac{\epsilon_\phi}{2}} + Z''\epsilon_\phi\right), \tag{19}$$

where the slow-roll parameters are defined in terms of ϕ as the following:

$$\epsilon_\phi = \frac{1}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_\phi = \frac{V''}{V}, \quad \kappa_\phi^2 = \frac{V'V'''}{V^2}. \tag{20}$$

Furthermore, Eqs. (17) and (18) can be derived with regard to ϕ resulting in:

$$N_* = \mathrm{sgn(V')}\int_{\phi_e}^{\phi_*}\frac{\mathrm{d}\phi}{\mathrm{Z}(\phi)\sqrt{2\epsilon_\phi}}, \tag{21}$$

$$\Delta_\mathcal{R} = \frac{1}{2\sqrt{3}\pi}\frac{V^{3/2}}{\sqrt{Z}|V'|}. \tag{22}$$

To calculate the numerical values of observables; the spectral index n_s, the tensor-to-scalar r, and the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$, we can write the number of e-folds N_* in the numerical form. Assuming a standard thermal history after inflation, one can write the number of e-folds N_* for the pivot scale $k_* = 0.002$ Mpc^{-1} in the following form (Liddle and Leach, 2003)

$$N_* \approx 64.7 + \frac{1}{2}\ln\frac{\rho_*}{M_{\rm P}^4} - \frac{1}{3(1+\omega_r)}\ln\frac{\rho_e}{M_{\rm P}^4} + \left(\frac{1}{3(1+\omega_r)} - \frac{1}{4}\right)\ln\frac{\rho_r}{M_{\rm P}^4}, \tag{23}$$

here, $\rho_e = (3/2)V(\phi_e)$ is the energy density at the end of inflation, ρ_r is the energy density at the end of reheating, and $\rho_* \approx V(\phi_*)$ is the energy density when the scale corresponding to k_* exits the horizon. ω_r is the equation of the state parameter during reheating. The definitions of ρ_r and ρ_* are given in the following forms

$$\rho_r = \left(\frac{\pi^2}{30}g_*\right)T_{reh}^4, \qquad \rho_* = \frac{3\pi^2\Delta_\mathcal{R}^2 r}{2}, \tag{24}$$

where T_{reh} corresponds to the reheat temperature; the temperature at which the universe is in thermal equilibrium and radiation dominates, here are the detailed studies related to the concept and its constraints (Bostan et al., 2018; Bostan, 2024). The standard model value $g_* = 106.75$, which gives the number of relativistic degrees of freedom, can be utilized to compute ρ_r.

In this work, we consider two different cases for the number of e-folds N_* which is defined in Eq. (23):

- First case: We take $w_r = 1/3$, it is the instant reheating assumption. With this assumption, the number of e-folds N_* in Eq. (23) reduces to the following form

$$N_* \approx 64.7 + \frac{1}{2} \ln \frac{\rho_*}{M_\mathrm{P}^4} - \frac{1}{4} \ln \frac{\rho_e}{M_\mathrm{P}^4}. \tag{25}$$

  Equation (25) demonstrates that inflationary predictions should not depend on T_{reh} regarding the instant reheating assumption.

- Second case: We take $w_r = 0$, and with this selection, the number of e-folds N_* should depend on the reheat temperature. In our numerical calculations, we take $T_{reh} = 10^{16}$ GeV and $T_{reh} = 10^8$ GeV in the following section. It is important to note that here, in the second case, the value of the e-folds number is less than the ones for the case when $w_r = 1/3$ (instant reheating). By taking $w_r = 0$, Eq. (23) becomes

$$N_* \approx 64.7 + \frac{1}{2} \ln \frac{\rho_*}{M_\mathrm{P}^4} - \frac{1}{3} \ln \frac{\rho_e}{M_\mathrm{P}^4} + \frac{1}{12} \ln \frac{\rho_r}{M_\mathrm{P}^4}. \tag{26}$$

It is clear that the second case depends on the reheat temperature. Equation (26) can be calculated by taking different values of T_{reh} so that one can see the relationship between reheat temperature and the number of e-folds N_*, as well as its effects on inflationary predictions. Throughout the next section, we will present the inflationary predictions of the Einstein frame β-exponential potential with minimal coupling in Palatini R^2 gravity assuming the standard thermal history after inflation. We first present our results analytically with rough approximations as an example, as well as we show the inflationary predictions numerically for both first (instant reheating) and second ($T_{reh} = 10^8$ GeV and $T_{reh} = 10^{16}$ GeV) cases for the number of e-folds, N_* that we have defined.

3. Results and Discussion

In this section, we begin by expressing the canonically normalized field (ζ) with respect to the original scalar field (ϕ). By using Eq. (7), the canonical scalar field $\zeta(\phi)$ can be found for the β-exponential inflation which is given in Eq. (12) as follows

$$d\zeta = \frac{d\phi}{\sqrt{1 + \frac{4\alpha V_0}{M_\mathrm{P}^4}\left(1 - \lambda\beta\frac{\phi}{M_\mathrm{P}}\right)^{1/\beta}}}, \tag{27}$$

to solve this differential equation, integrate both sides:

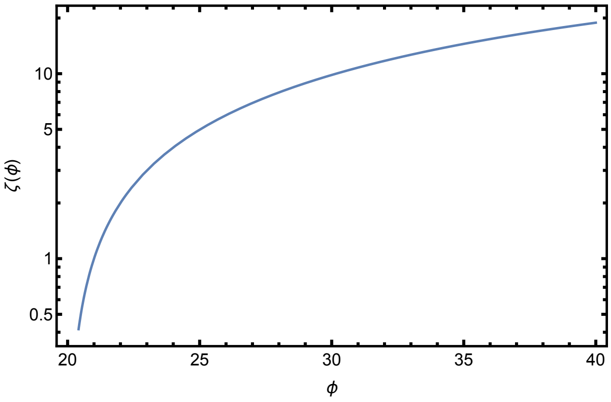

Figure 2: Plot of ϕ vs. $\zeta(\phi)$. We set the values as follows: $V_0 = 10^{-9}$, $\alpha = 10^8$, $\lambda = 0.1$, $\beta = 0.5$ (M_P is set to 1).

$$\zeta(\phi) = \int \frac{d\phi}{\sqrt{1 + \frac{4\alpha V_0}{M_\mathrm{P}^4}\left(1 - \lambda\beta\frac{\phi}{M_\mathrm{P}}\right)^{1/\beta}}}. \tag{28}$$

Let $\gamma \equiv 1 - \lambda\beta\frac{\phi}{M_\mathrm{P}}$, then

$$d\gamma = -\frac{\lambda\beta}{M_\mathrm{P}} d\phi,$$

and then the integral becomes:

$$\zeta(\gamma) = -\frac{M_\mathrm{P}}{\lambda\beta} \int \frac{d\zeta}{\sqrt{1 + \frac{4\alpha V_0}{M_\mathrm{P}^4}\gamma^{1/\beta}}}. \tag{29}$$

This integral can be evaluated in terms of a Hypergeometric function. We obtain the final result by inserting $\gamma \equiv 1 - \lambda\beta\frac{\phi}{M_\mathrm{P}}$ as follows

$$\zeta(\phi) = \frac{(\beta\lambda\phi - M_\mathrm{P})\, {}_2F_1\left(\frac{1}{2}, \beta; \beta + 1; -\frac{4V_0\alpha\left(1 - \frac{\phi\beta\lambda}{M_\mathrm{P}}\right)^{1/\beta}}{M_\mathrm{P}^4}\right)}{\beta\lambda}, \tag{30}$$

where ${}_2F_1(a; b; c; z)$ is the Hypergeometric function. In order to illustrate the behavior of the function of $\zeta(\phi)$, we compute it numerically for various choices of ϕ, and we take into consideration the observables constraints as mentioned in Table 1. The resultant plot is given in Fig. 2. As it is the general case that inflationary potentials are complex to compute their interval, and consequently calculate the potential in terms of $\zeta(\phi)$, the way to find the interval of Eq. (30) is through numerical techniques.

Next, we show our results for the slow-roll analysis. Throughout the subsequent analysis results in this section, M_P will be set unity. By using Eq. (19), the slow-roll parameters, $\epsilon(\phi_*)$ and $\eta(\phi_*)$, can be found in the Einstein frame of a minimally coupled β-exponential potential in Palatini R^2 gravity, which is defined in Eq. (14) as follows,

$$\epsilon(\phi_*) \simeq \frac{\lambda^2}{8\alpha V_0 x^{\frac{1}{\beta}+2} + 2x^2},$$
$$\eta(\phi_*) \simeq \frac{\lambda^2\left(-2\beta + \frac{3}{4\alpha V_0 x^{1/\beta+1}} - 1\right)}{2x^2}, \tag{31}$$

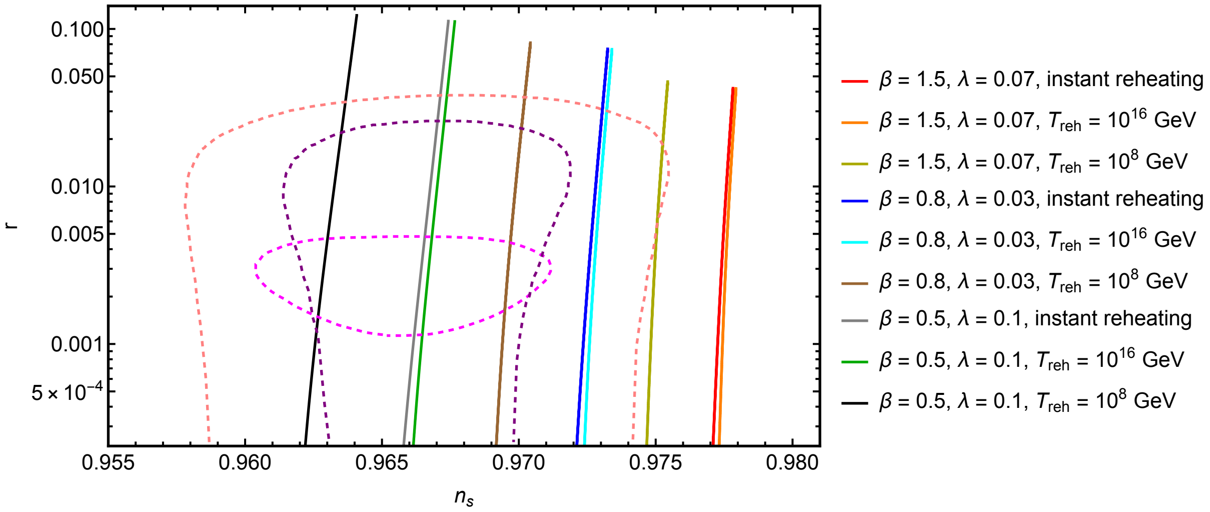

Figure 3: $n_s - r$ predictions for the selected parameters of minimally coupled β-exponential inflation with an R^2 term in Palatini formalism, α varies in the range of $[10^7 - 10^{15}]$. Pink(purple) dashed contours indicate the recent 95%(68%) CL given by BICEP/Keck (Ade et al., 2021), while the magenta dashed line corresponds to the sensitivity forecast for the future CMB-S4 (Abazajian et al., 2019).

where $1 - \beta\lambda\phi_* \equiv x$. Also, by utilizing Eq. (16), the main observational parameters, the spectral index n_s and the tensor-to-scalar ratio r, can be acquired analytically for the β-exponential potential as follows

$$n_s(\phi_*) \simeq 1 - \frac{(2\beta+1)\lambda^2}{x^2}, \quad r(\phi_*) \simeq \frac{8\lambda^2}{x^2\left(4\alpha V_0 x^{1/\beta} + 1\right)}. \quad (32)$$

Regarding Eq. (32), it is important to mention that for the β-exponential potential in minimal coupling with an R^2 term in Palatini formalism, even though this is not the case for the spectral index n_s predictions, tensor-to-scalar ratio r depends on the α parameter significantly, which is confirmed from our numerical results shown in Table 1, and related more analysis can be obtained through studying Figure 7 in a specific range of α values, we will elaborate on this point during the related figures. From the analytical results that are given by Eq. (32), it can be mentioned that the predictions of the tensor-to-scalar ratio r should decrease as the α parameter increases. In addition, the number of e-folds N_* is obtained by using Eq. (21) for our inflationary model in the following form

$$N_* \simeq \frac{\phi_*(\beta\lambda\phi_* - 2)}{2\lambda}. \quad (33)$$

One can write the spectral index n_s and the tensor-to-scalar ratio r expressions given in Eq. (32) in terms of the number of e-folds N_* for the β-exponential potential by considering different kinds of approximations. For instance, let us assume $\beta\phi_*^2/2 \gg \phi_*/\lambda$ in Eq. (33), then we can obtain $x \approx 1 \mp (\lambda\sqrt{2\beta N_*})$. If one inserts this into the equation (32), the predictions can be acquired in terms of the number of e-folds N_* as follows

$$n_s \simeq 1 - \frac{(2\beta+1)\lambda^2}{\left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^2},$$
$$r \simeq \frac{8\lambda^2}{\left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^2\left(4\alpha V_0\left(1 \mp (\lambda\sqrt{2\beta N_*})\right)^{1/\beta} + 1\right)}. \quad (34)$$

In addition, for the case of $\beta\lambda\phi_* \ll 1$, one can find $x \approx 1 + \lambda^2\beta N_*$. With this approximation, the spectral index n_s and the tensor-to-scalar ratio r can be obtained in terms of the number of e-folds N_* analytically as follows

$$n_s \simeq 1 - \frac{(2\beta+1)\lambda^2}{(1+\lambda^2\beta N_*)^2},$$
$$r \simeq \frac{8\lambda^2}{(1+\lambda^2\beta N_*)^2\left(4\alpha V_0(1+\lambda^2\beta N_*)^{1/\beta} + 1\right)}. \quad (35)$$

From Figure 3, our examination takes into consideration different options of the reheating scenarios in order to have a better image, which leads to a both better and accurate analysis. We take the highest $T_{reh} = 10^{16}$ GeV, the value that inflation models can reach, which is the GUT scale, as well as we take lower value, $T_{reh} = 10^8$ GeV to show the differences of reheating effects on inflationary predictions. Moreover, one can relate the consistency of the plot to the equations derived and presented in this work. As ϵ decreases, one can see that the predicted values of the tensor-to-scalar ratio r decrease as well, and this leads to lines with different slopes. The positions of the lines in this plot imply the sensitivity of the tensor-to-scalar ratio r and the spectral index n_s to parameters as β, λ, and the reheating scenarios. Comparing the constrained predictions obtained from our model to the observational data, one can spot how our model fits the data right and well, especially given that the solid lines approach the best-fit regions of the contours, which makes our model viable.

Furthermore, ref. (dos Santos et al., 2022) examined the β-exponential inflationary model for both minimally and non-minimally coupled scalar fields with gravity. They showed a $n_s - r$ plane for both cases. In particular, for the minimally coupled case, they found their model predictions of $n_s - r$ are in good agreement at 2σ CL when using Planck 2015 data but for the most recent data from Planck 2018 + BAO measurements, the agreement between their results and the recent data

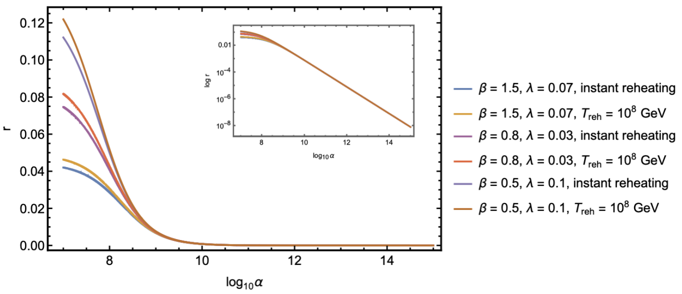

Figure 4: $r - \alpha$ plane for the selected parameters of minimally coupled β-exponential inflation with an R^2 term in Palatini formalism.

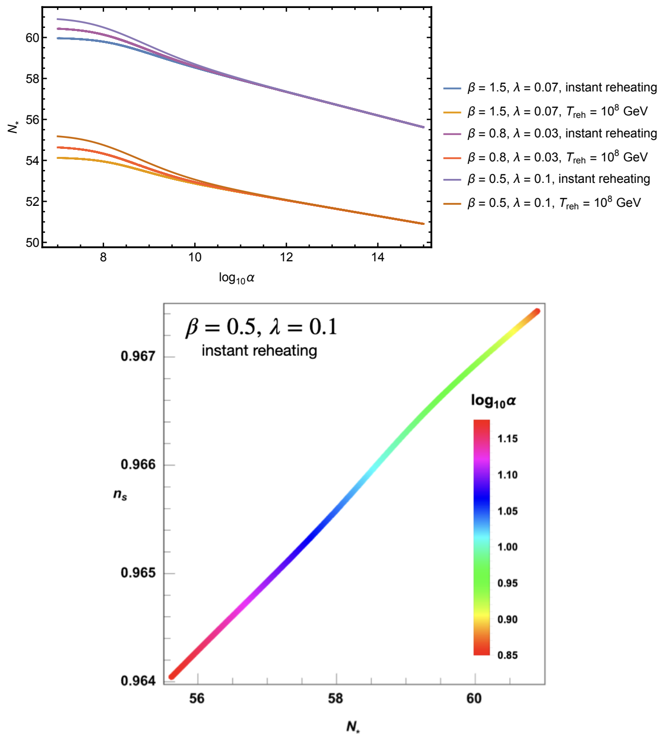

Figure 5: For the minimally coupled β-exponential inflation with an R^2 term in Palatini formalism, the top panel depicts $N_* - \alpha$ plane for the selected parameters, as well as bottom panel shows how the $N_* - n_s$ predictions change depending on the α parameter with $\beta = 0.5$ and $\lambda = 0.1$ for the instant reheating assumption.

is lost. On the contrary, our results in this work show that the inflationary predictions for the β-exponential inflation with minimally coupled in Palatini R^2 gravity can be within the recent Planck + BICEP/Keck data even in 1σ CL region. Also, for the non-minimal coupling case, ref. (dos Santos et al., 2022) indicated the agreement between their results and recent cosmological data for some of the selected non-minimal coupling parameters. One can see from Figure 3, the results of the inflationary predictions are very close to each other for the instant reheating and $T_{reh} = 10^{16}$ GeV cases, for the following figures we will precede by only showing our results regarding two different cases: $T_{reh} = 10^8$ GeV and the instant reheating scenario in order to avoid overlapping of the curves and overcrowding in the figures.

In addition, Figure 4 depicts the relationship between the tensor-to-scalar ratios r and the parameter α for different values of β, λ and the reheat temperature cases T_{reh}. It can be noticed from this figure that the tensor-to-scalar ratio r decreases as α values increase for all sets of β, λ, and T_{reh}. This kind of behavior is consistent with the expectation that for larger α values, we can spot more suppressed tensor modes, resulting in lower tensor-to-scalar ratio r values. In addition, ref. (Tenkanen, 2019) studied the minimal Higgs inflation with an R^2 term in Palatini formalism, the study found the predictions following: $n_s \simeq 0.941$, $r \simeq 0.3/(1 + 10^{-8}\alpha)$ for $N_* \simeq 50$. For instance, they calculated the inflationary predictions, for $\alpha = 10^{16}$ and $N_* = 56 \rightarrow n_s \sim 0.947$, $r \sim 8 \times 10^{-10}$, as well as for $\alpha = 10^8$ and $N_* = 56 \rightarrow n_s \sim 0.947$, $r \sim 0.06$. Regarding their results, as α increases, r decreases notably but there are no remarkable changes on n_s predictions regarding α values. Our results in this study are in good agreement with ref. (Tenkanen, 2019). On the other hand, for lower values of β, in a specific choice of α, we can notice higher values of the tensor-to-scalar ratio r as well, reflecting the sensitivity of the inflationary dynamics to this mentioned parameter. Additionally, we cannot ignore the impact of the reheating temperature T_{reh} choice or case as well, for that in some given β and λ and instant reheating scenario leads to slightly different tensor-to-scalar ratio r values compared to the scenario of $T_{reh} = 10^8$ GeV. This difference can be interpreted due to the interplay between the reheating phase and the dynamics of the scalar field ϕ during inflation.

Also, Figure 4 shows that for lower values of β, the tensor-to-scalar ratio r tends to be higher *an inverse relation*; however, it has a positive correlation with the given values of λ. This type of behavior of the curves aligns with what one can expect that larger deviations from the standard inflationary potential reflect a higher tensor-to-scalar ratio r. Furthermore, the curves also illustrate the sensitivity of the tensor-to-scalar ratio r to the post-inflationary reheating phase, which is captured by different values of the number of e-folds N_*, and this is very important since it points out to the fact that the inflationary predictions depend on the dynamics of the scalar field ϕ, the slow-roll parameters, and the reheating phase.

For Figure 5, we can comment with the following. The top panel presents the relationship between the number of e-folds N_* and the parameter α for selected values of β and λ. The figure shows that as the parameter α increases, the number of e-folds N_* decreases for each set of parameters, indicating that a stronger R^2 term reduces the duration of inflation. This reduction is more pronounced for lower β values, reflecting the dependence of the inflationary dynamics on the shape of the potential, as β controls the steepness of the potential. The effect of the dimensionless parameter α in the top plot is subtle and does not play a major role when one compares its relation with the slow-roll parameters, which is stronger as we can notice from the other figures (e.g., Fig. 4), and this explains why there is no direct and obvious dependence of the number of e-folds on the parameter α in Eq. 33. Hence this shows the alignment between our analytical and numerical results, which makes our paper more viable. The bottom panel examines how the spec-

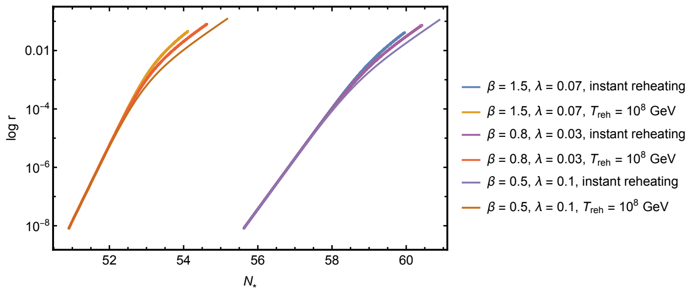

Figure 6: $r - N_*$ plane for the selected parameters of minimally coupled β-exponential inflation with an R^2 term in Palatini formalism.

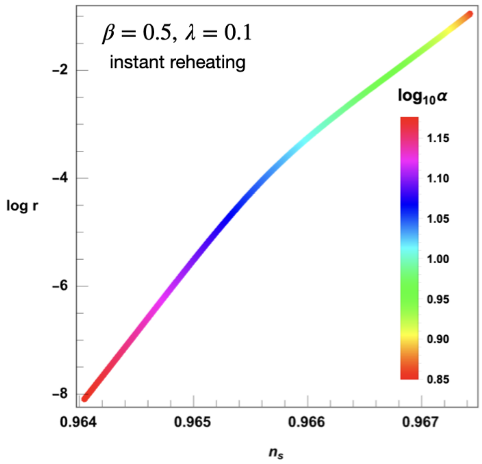

Figure 7: The plot depicts how the $n_s - r$ predictions change depending on the α parameter for the β-exponential inflation with an R^2 term in Palatini formalism with $\beta = 0.5$ and $\lambda = 0.1$ for the instant reheating assumption.

tral index n_s varies with the number of e-folds N_* for the fixed values of $\beta = 0.5$ and $\lambda = 0.1$, under the assumption of instant reheating. The color gradient represents different values of the logarithmic scale of α, illustrating that as the number of e-folds N_* increases, the spectral index n_s also increases slightly.

Moreover, Figure 6 represents the tensor-to-scalar ratio r as a function of the number of e-folds N_* for various sets of β and λ. Different curves correspond to different reheating scenarios. The curves generally show a decreasing trend of the tensor-to-scalar ratio r as the number of e-folds N_* increases. For the higher number of e-folds N_* values, the tensor-to-scalar ratio r tends to stabilize, showing less variation, which is consistent with the expectation that as the inflationary phase progresses, the contributions to the tensor-to-scalar ratio r diminish. The comparison between instant reheating and reheating at $T_{reh} = 10^8$ GeV illustrates that lower reheating temperatures generally lead to slightly lower values of the tensor-to-scalar ratio r for the same number of e-folds N_*. This is consistent with the expectation that a longer reheating phase (lower T_{reh}) allows for additional red-shifting of the tensor modes, reducing the tensor-to-scalar ratio r.

In Figure 7, we took advantage of the color gradient technique to indicate the continuous dependence of the tensor-to-scalar ratio r on both the spectral index n_s and α provides us with a good alignment with what we already had expected analytically earlier in this section. From this figure, a positive correlation between the tensor-to-scalar ratio r and the spectral index n_s can be noticed, which is something typical of many inflationary models. However, as the spectral index n_s gets larger, the curve starts to behave in a more flat manner, and this can indicate that the sensitivity of the change of the tensor-to-scalar r with respect to the spectral index n_s becomes less sensitive to changes in α. One can refer to this last sentence as output to the slow-roll parameters ϵ and η stabilizing in this regime, which is something that can be expected. Looking at the ranges of the logarithmic scale of the tensor-to-scalar ratio $\log_{10} r$, which is produced by our model, we can see their agreement with the observational data.

Lastly, we also show our results in Table 1, which is a comprehensive table regarding three different parameters of α, λ, and β, and the results presented here have been checked by running the calculations through various numerical techniques and algorithms in order to obtain a very valid set of results. From this table, we can mention that for the selected values of e-folds, $45, 55$, and 61, the spectral index n_s predictions remain similar for the larger and smaller values of α parameter, but the change in the r values is quite large which is something we expect after observing our both analytical results and from Figure 7. Thus, it can be concluded that the β-exponential inflation with an R^2 term in Palatini formalism can be aligned and in good agreement with the recent cosmological data for the larger α values, which makes the inflationary model compatible with the data. We can see that as the parameter α value increases, the tensor-to-scalar ratio r value becomes very tiny. In addition, from our results, we can mention that the spectral index n_s values generally show a change depending on reheating scenarios. It is pivotal to note that here for the larger number of e-folds N_*, larger spectral index n_s values are obtained. Also, the predictions of the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$ do not alter so much with the free parameter α values. Additionally, we find the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$ predictions are very tiny to be observed at least in the near future.

4. Summary and conclusions

In this work, we have studied the minimally coupled β-exponential inflation with an R^2 term in Palatini formalism. Before we start into our analysis and plotting process we have provided a regressive and well-detailed mathematical framework for our paper in order to have the best viable results possible. For this scenario, we calculate the inflationary predictions thoroughly, as well as compare them with the recent cosmological data and future sensitivity forecast by CMB-S4. Additionally, this work points out to the fact that in the future through observational data, stronger constraints might be applied resulting in ruling out most of the existing inflationary models, for the case of our model, Figure 6 gives us a promising results that this model will thrive even in the existence of much more strong constraints.

Moreover, the analysis provided in our study incorporates a detailed study of the reheating dynamics on inflationary observ-

Table 1: The inflationary parameter sets of approximate values for the minimally coupled β-exponential inflation with an R^2 term in Palatini formalism.

N_*	$n_s(\phi_*)$	$r(\phi_*)$	$\mathrm{d}n_s/\mathrm{d}\ln k\ (\phi_*)$	ϕ_*	ϕ_e	V_0
$\alpha = 10^8, \lambda = 0.1, \beta = 0.5$						
61	0.967	0.049	-5.29×10^{-4}	35.68	21.40	6.58×10^{-9}
55	0.964	0.051	-6.46×10^{-4}	34.91	21.40	8.04×10^{-9}
45	0.956	0.055	-9.64×10^{-4}	33.49	21.39	1.20×10^{-8}
$\alpha = 10^{15}, \lambda = 0.1, \beta = 0.5$						
61	0.967	8.04×10^{-9}	-5.34×10^{-4}	35.64	20.07	6.65×10^{-9}
55	0.964	8.04×10^{-9}	-6.47×10^{-4}	34.91	20.07	8.05×10^{-9}
45	0.956	8.04×10^{-9}	-9.64×10^{-4}	33.49	20.06	1.20×10^{-8}

ables, which are pivotal for the study. We investigate a variety of reheating temperatures, and we examine their influence on the inflationary predictions, particularly the spectral index n_s, and the tensor-to-scalar ratio r. Additionally, we have shown that the inclusion of reheating impacts gives us a clearer information for the effects of the reheating on the predictions, which does not only refine our model's predictions for the inflationary models but also it aligns our results closely with the current observational constraints from current data, and makes it sensitive for future constraints forecast by CMB-S4.

We have found that our scenario can be aligned and in good agreement with the recent cosmological data for larger α values, which makes our model consistent with the data, taking very tiny r values, reaching $r \sim 10^{-9}$. This numerical result is in good agreement with the results we found analytically for this scenario. In addition, this result is consistent with the studies in the literature as we discussed in the previous section. We have also discussed our results by considering different reheat scenarios. We observe that depending on the values of reheat temperature, the difference appears in the spectral index n_s predictions for the selected parameters in our inflationary model. In addition, we show the predictions of the running of the spectral index $\mathrm{d}n_s/\mathrm{d}\ln k$ are too small for our scenario.

Lastly, it is important to note that the studies in literature for the β-exponential inflation models that are taking into consideration the different scenarios and gravity models are still ongoing, thus it is considered that our results in this work will be very important to be depicted in order to build a bridge between different inflationary scenarios, gravity theories and braneworld cosmological frameworks regarding this type of potential model. The β-exponential potential models can be obtained through the braneworld scenario framework, and considering these kinds of potential models within the context of the braneworld scenarios are strongly motivated for the primordial inflation.

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